$$f : [0, 2\pi] \to \mathbb{R}^2$$
$$x \mapsto \begin{pmatrix} \cos x \\ 1 - e^x \end{pmatrix}$$

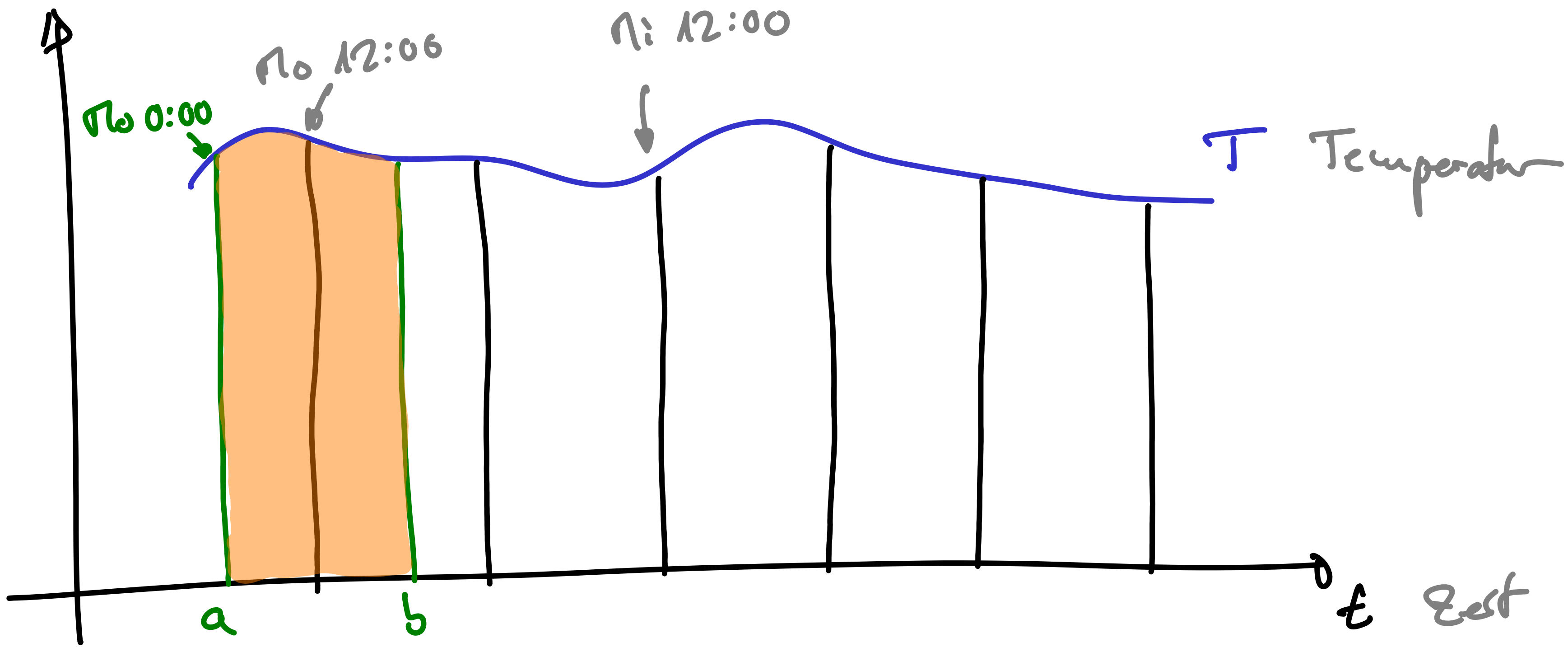
ges: F mit $F' = f$

$$F : [0, 2\pi] \to \mathbb{R}^2$$
$$x \mapsto \begin{pmatrix} \sin x \\ x - e^x \end{pmatrix}$$

weitere Stammfkt ist z.B.

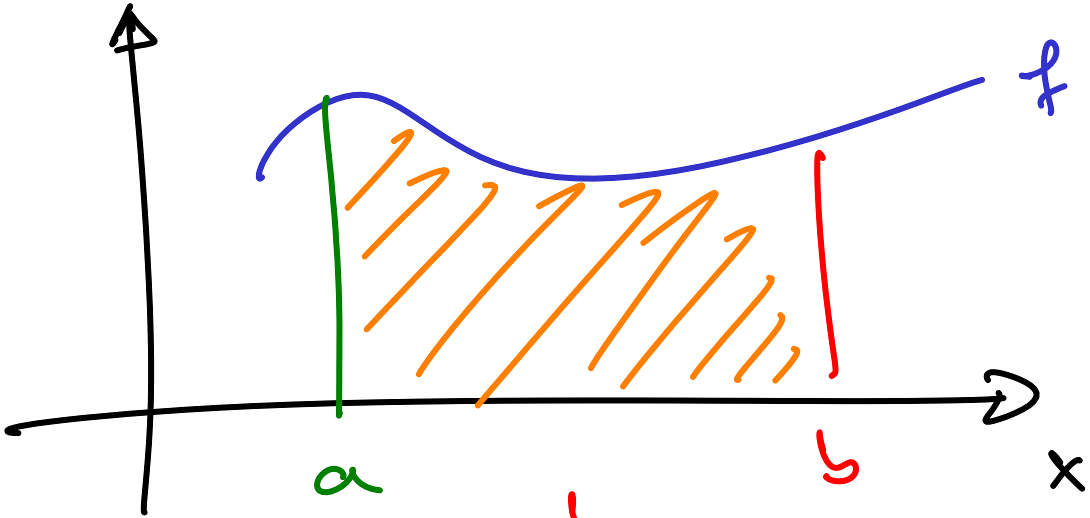
$$\tilde{F}(x) = \begin{pmatrix} \sin x + 2014 \\ x - e^x - \pi \end{pmatrix} \quad \left(\text{hier } C = \begin{pmatrix} 2014 \\ -\pi \end{pmatrix} \right)$$

da $\tilde{F}'(x) = f(x)$

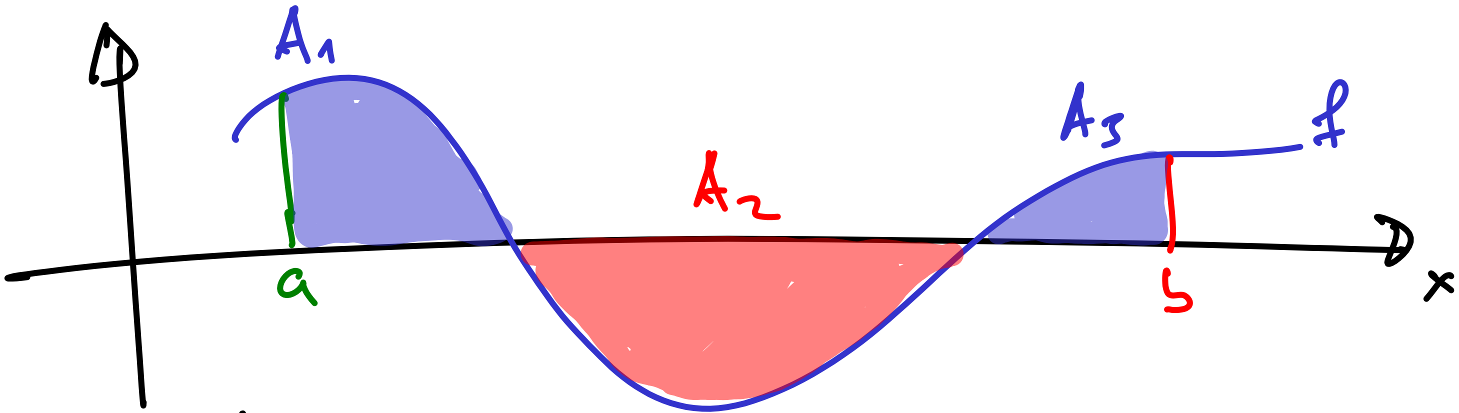


24 h-Mittel $= \frac{\text{Fläche}}{24\,h}$

Fläche $= \int_a^b T(t)\,dt$

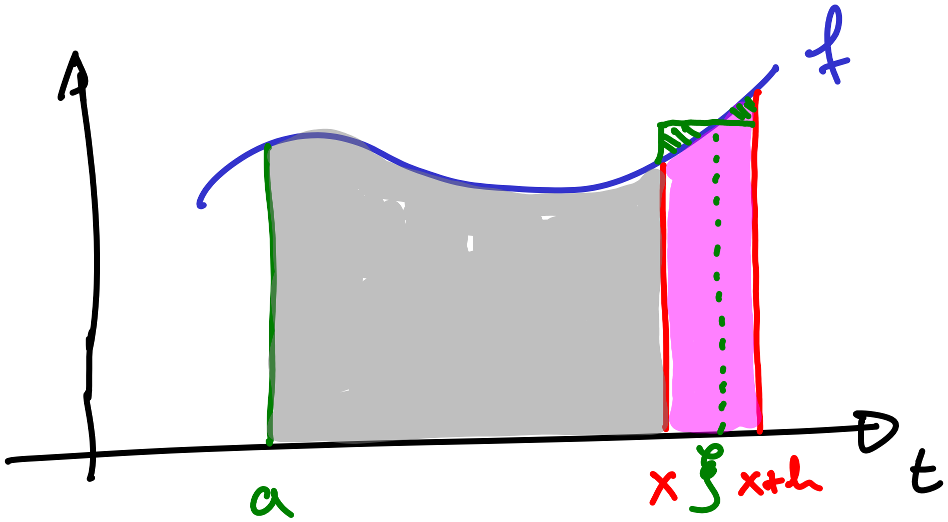


$$\text{Fläche} = \int_a^b f(x)\, dx$$



$$\int_a^b f(x)\, dx = A_1 - A_2 + A_3$$

Beweisidee zum HS



$$F(x) = \int_a^x f(t)\,dt$$

z.Z.: F ist Stammfkt. von f

also $F' = f$

$$F'(x) = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(\xi)\cdot \cancel{h}}{\cancel{h}}$$

es gibt ein $\xi \in [x, x+h]$

$$= f(x)$$

$h \to 0 \Rightarrow \xi \to x$

$$\int_0^1 \left(2x^5 - 7x^2 + 3\right) dx$$

$$= \left[2\frac{x^6}{6} - 7\frac{x^3}{3} + 3x\right]_0^1$$

Schreibweise für

$$= \left(\frac{1}{3}\cdot 1^6 - \frac{7}{3}\cdot 1^3 + 3\cdot 1\right) - \left(\frac{1}{3}\cdot 0^6 - \frac{7}{3}\cdot 0^3 + 3\cdot 0\right)$$

$$= \frac{1}{3} - \frac{7}{3} + 3 - 0 = 1$$

$$\int_1^3 \frac{1}{x^2}\, dx = \int_1^3 \frac{dx}{x^2} = \int_1^3 x^{-2}\, dx = \left[-x^{-1}\right]_1^3 = \left[-\frac{1}{x}\right]_1^3$$

$$= -\frac{1}{3} - (-1) = \frac{2}{3}$$

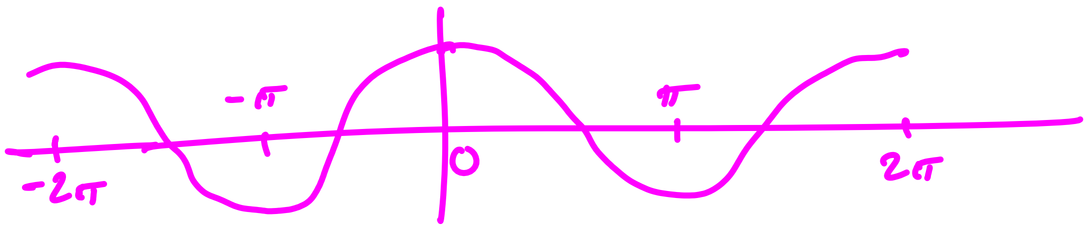
$$\int_1^3 \frac{dx}{x} = \left[\log x\right]_1^3 = \log 3 - \underbrace{\log 1}_{=0} = \log 3$$

$n \in \mathbb{Z}$

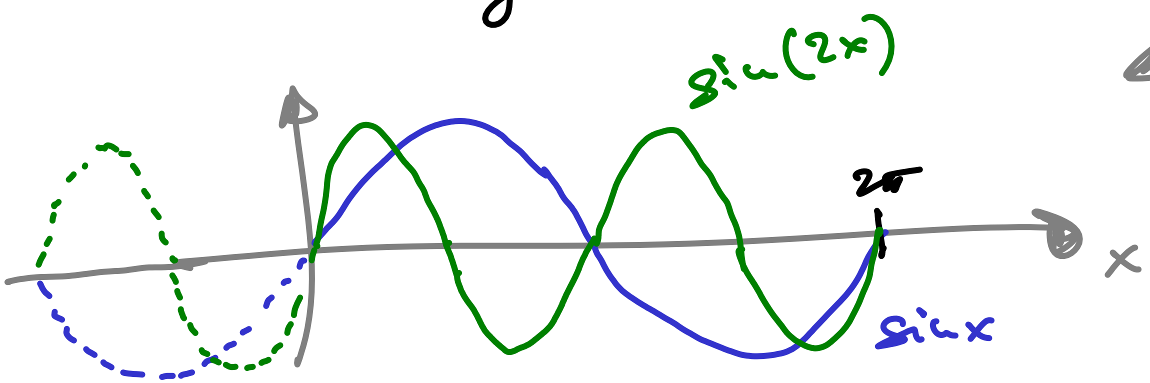
$$\int_{-\pi}^{\pi} \sin(nx)\,dx$$

$$= \left[-\cos(nx)\cdot\frac{1}{n}\right]_{-\pi}^{\pi} = -\frac{1}{n}\left[\cos(nx)\right]_{-\pi}^{\pi}$$

$$= -\frac{1}{n}\Big(\cos(n\pi) - \cos(-n\pi)\Big) = -\frac{1}{n}\Big((-1)^n - (-1)^n\Big) = 0$$



ohne Rechnung



Begründung für

$$\int_0^{2\pi} \sin(nx)\,dx = 0$$

$$\int_{-\pi}^{\pi} \sin^2(nx)\,dx = \int_{-\pi}^{\pi} \left(\sin(nx)\right)^2 dx$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$ Add. Th. des cos

$$= 1 - 2\sin^2 x$$

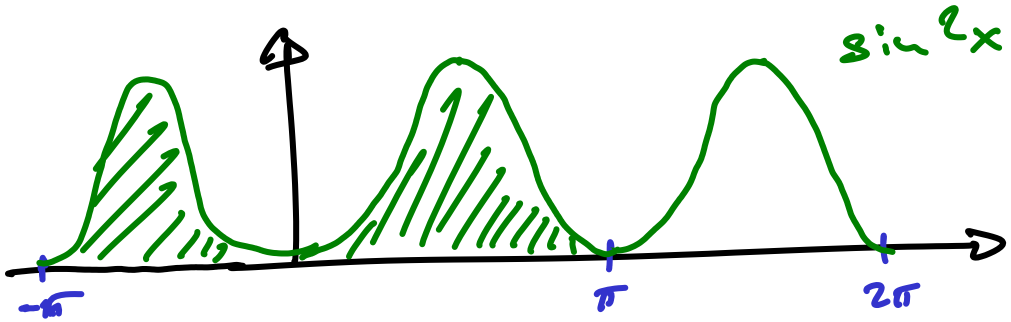
Pyth. $\cos^2 x + \sin^2 x = 1$

$$\Rightarrow \quad \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\int_{-\pi}^{\pi} \sin^2(nx)\,dx = \int_{-\pi}^{\pi} \left(\frac{1}{2} - \frac{1}{2}\cos(2nx)\right) dx$$

$$= \left[\frac{1}{2}x - \frac{1}{4n}\sin(2nx)\right]_{-\pi}^{\pi}$$

$$= \left(\frac{\pi}{2} - 0\right) - \left(-\frac{\pi}{2} - 0\right) = \pi$$



Produktregel: $(f(x)\,g(x))' = f'(x)\,g(x) + f(x)\,g'(x)$

$\int_a^b \ldots \, dx \quad \Longrightarrow$

$$\left[f(x)\,g(x)\right]_a^b = \underline{\int_a^b f'(x)\,g(x)\,dx} + \int_a^b f(x)\,g'(x)\,dx$$

$$\Longleftrightarrow \quad \underline{\int_a^b f'(x)\,g(x)\,dx} = \left[f(x)\,g(x)\right]_a^b - \int_a^b f(x)\,g'(x)\,dx$$

$$\int_0^{\pi/2} \underset{g}{x} \cdot \underset{f'}{\cos x}\, dx = \left[x \sin x\right]_0^{\pi/2} - \int_0^{\pi/2} 1 \cdot \sin x \, dx$$

$$= \frac{\pi}{2} \cdot 1 - 0 \cdot 0 - \left[-\cos x\right]_0^{\pi/2}$$

$$= \frac{\pi}{2} + \underbrace{\cos\left(\frac{\pi}{2}\right)}_{=0} - \underbrace{\cos(0)}_{=1} = \frac{\pi}{2} - 1$$

$$\int \underset{f'}{1} \cdot \underset{g}{\log x}\, dx = x \cdot \log x - \int \underbrace{x \cdot \frac{1}{x}}_{=1} dx = x \cdot \log x - x$$

Übrigens:

Test $\left(x \log x - x\right)' = \log x + x \cdot \frac{1}{x} - 1 = \log x$ ☺

$$n \neq m, \quad n, m \in \mathbb{N}_0$$

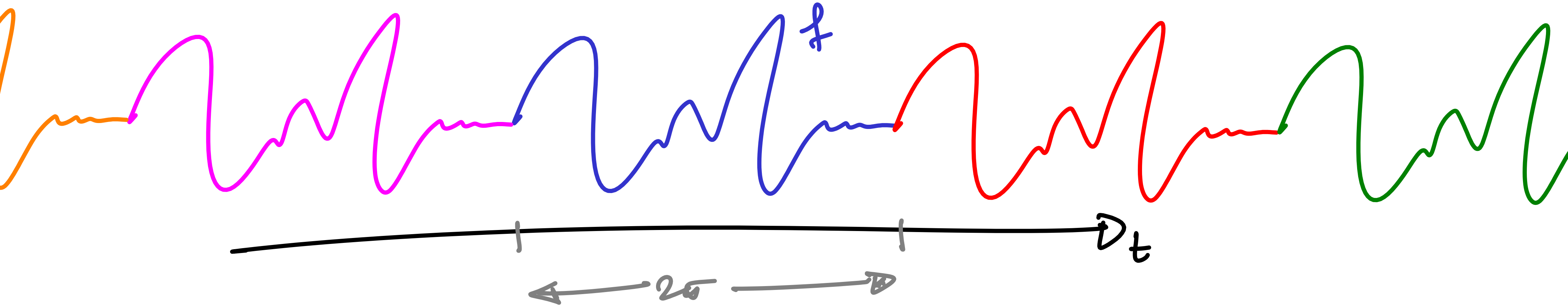
$$\int_{-\pi}^{\pi} \underset{f'}{\sin(nx)}\, \underset{g}{\sin(mx)}\, dx = \underbrace{\left[- \underset{f}{\frac{\cos(nx)}{n}}\, \underset{g}{\sin(mx)} \right]_{-\pi}^{\pi}}_{=0} + \int_{-\pi}^{\pi} \underset{f}{\frac{\cos(nx)}{n}}\, \underset{g'}{\cos(mx) \cdot m}\, dx$$

$$= \frac{m}{n} \int_{-\pi}^{\pi} \underset{f'}{\cos(nx)}\, \underset{g}{\cos(mx)}\, dx$$

$$= \frac{m}{n} \underbrace{\left[\frac{\sin(nx)}{n} \cos(mx) \right]_{-\pi}^{\pi}}_{=0} + \frac{m}{n} \int_{-\pi}^{\pi} \frac{\sin(nx)}{n} \sin(mx) \cdot m \, dx$$

$$= \frac{m^2}{n^2} \int_{-\pi}^{\pi} \sin(nx) \sin(mx)\, dx$$

$$\Rightarrow \underbrace{\left(1 - \frac{m^2}{n^2}\right)}_{\neq 0} \cdot \underbrace{\int_{-\pi}^{\pi} \sin(nx) \sin(mx)\, dx}_{=0} = 0$$



so eine Fkt. ist Summe von sin- & cos-Termen

$$\int_{-\pi}^{\pi} f(t)\sin(mt)\,dt = \underbrace{\underbrace{\int_{-\pi}^{\pi} a_0 \sin(mt)\,dt}_{=0} + \underbrace{\sum_n \int_{-\pi}^{\pi} a_n \cos(nt)\sin(mt)\,dt}_{=0\ (\text{ÜA})} + \underbrace{\int_{-\pi}^{\pi} b_n \sin(nt)\sin(mt)\,dt}_{=\begin{cases}\pi b_n, & n=m\\ 0, & n\neq m\end{cases}}}_{=\pi b_m}$$