

$$f: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$x \mapsto \begin{pmatrix} \cos x \\ 1 - e^x \end{pmatrix}$$

ges: F mit $F' = f$

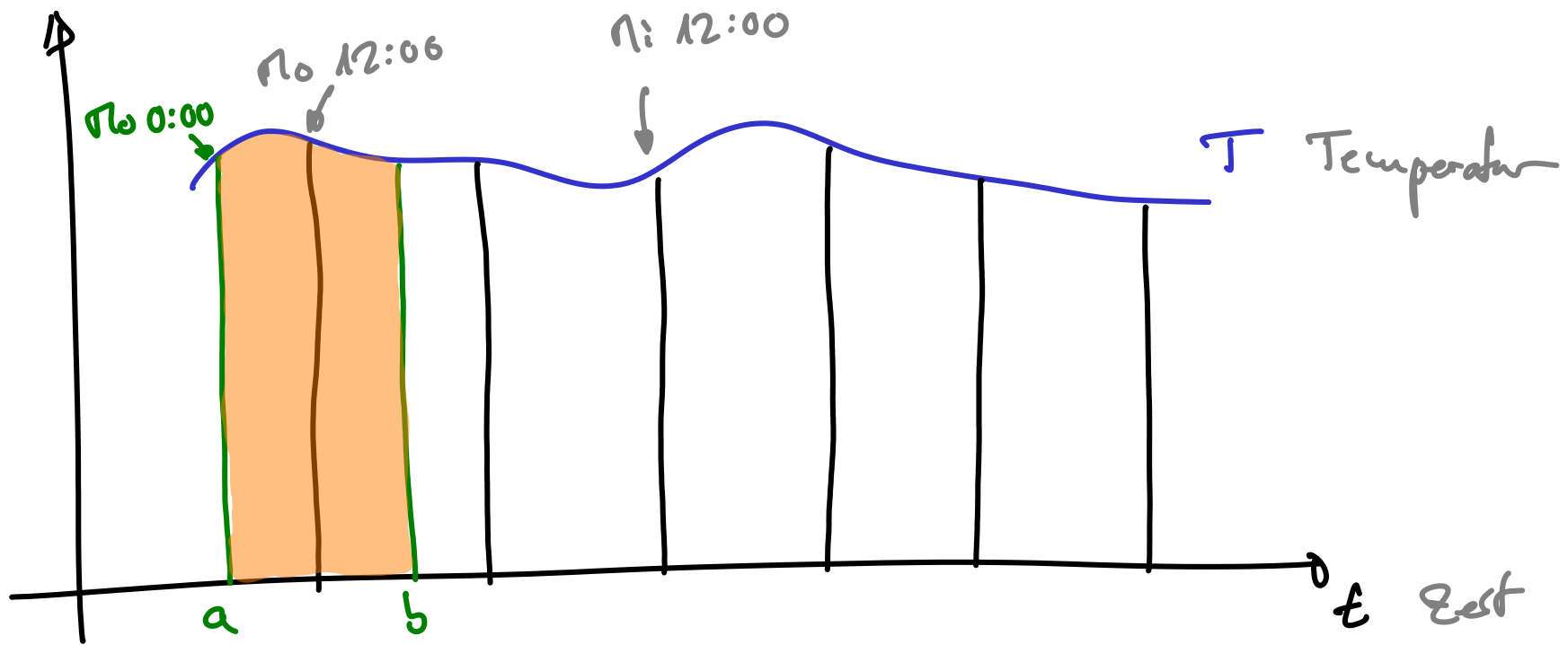
$$\tilde{F}: [0, 2\pi] \rightarrow \mathbb{R}^2$$

$$x \mapsto \begin{pmatrix} \sin x \\ x - e^x \end{pmatrix}$$

weitere Stammfkt ist z.B.

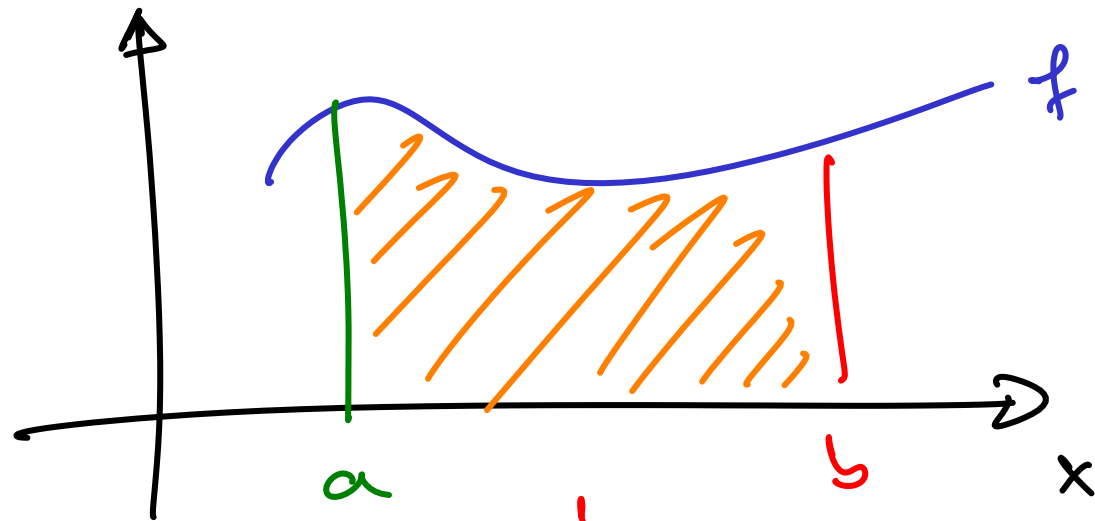
$$\tilde{\tilde{F}}(x) = \begin{pmatrix} \sin x + 2014 \\ x - e^x - \pi \end{pmatrix} \quad \left(\text{hier } C = \begin{pmatrix} 2014 \\ -\pi \end{pmatrix} \right)$$

$$\text{da } \tilde{\tilde{F}}'(x) = f(x)$$



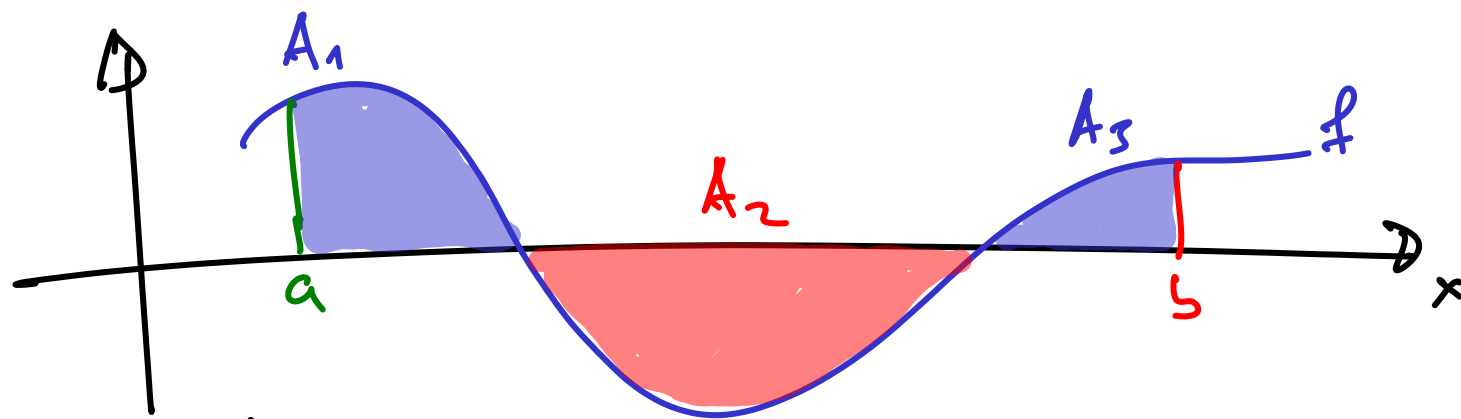
$$24 \text{ h-Nittel} = \frac{\text{Fläche}}{24 \text{ h}}$$

$$\text{Fläche} = \int_a^b T(t) dt$$



$$\text{Fläche} = \int_a^b f(x) dx$$

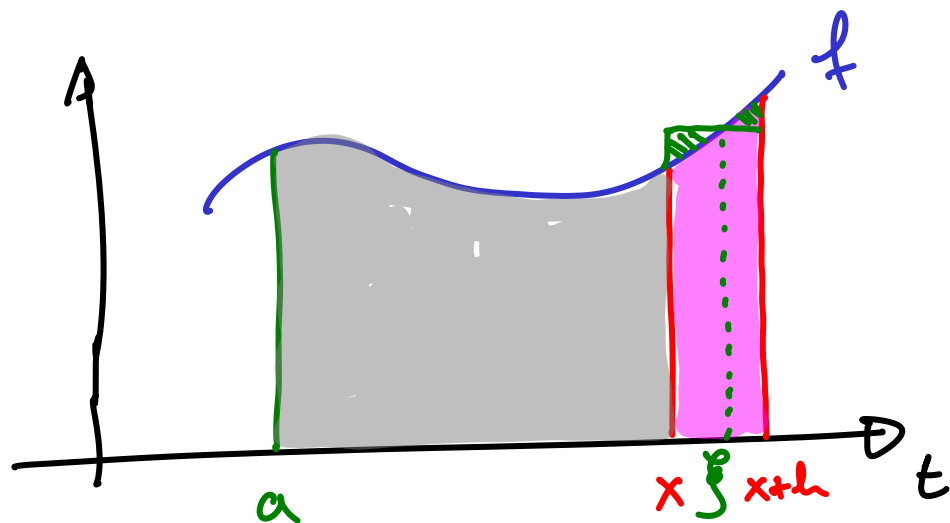
The integral is written in orange. The limits a and b are also in orange. The function $f(x)$ and the differential dx are in blue. Small grey arrows point upwards from the a and b in the denominator to the a and b in the numerator.



$$\int_a^b f(x) dx = A_1 - A_2 + A_3$$

The integral is written in blue. The limits a and b are also in blue. The function $f(x)$ and the differential dx are in black. The areas A_1 , A_2 , and A_3 are in blue, red, and blue respectively.

Beweisidee zum HS



$$F(x) = \int_a^x f(t) dt$$

z.Z.: F ist Stammfkt. von f

$$\text{also } F' = f$$

$$F'(x) = \lim_{h \rightarrow 0} \frac{F(x+h) - F(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(\xi) \cdot \cancel{h}}{\cancel{h}}$$

es gibt ein $\xi \in [x, x+h]$

$$= f(x)$$

$$h \rightarrow 0 \Rightarrow \xi \rightarrow x$$

$$\int_0^1 (2x^5 - 7x^2 + 3) dx$$

$$= \left[2 \frac{x^6}{6} - 7 \frac{x^3}{3} + 3x \right]_0^1$$

Schreibweise für

$$= \left(\frac{1}{3} \cdot 1^6 - \frac{7}{3} \cdot 1^3 + 3 \cdot 1 \right) - \left(\frac{1}{3} \cdot 0^6 - \frac{7}{3} \cdot 0^3 + 3 \cdot 0 \right)$$

$$= \frac{1}{3} - \frac{7}{3} + 3 - 0 = 1$$

$$\int_1^3 \frac{1}{x^2} dx = \int_1^3 \frac{dx}{x^2} = \int_1^3 x^{-2} dx = \left[-x^{-1} \right]_1^3 = \left[-\frac{1}{x} \right]_1^3$$

$$= -\frac{1}{3} - (-1) = \frac{2}{3}$$

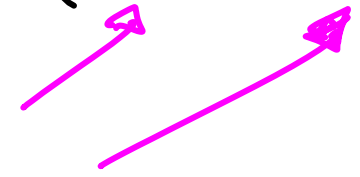
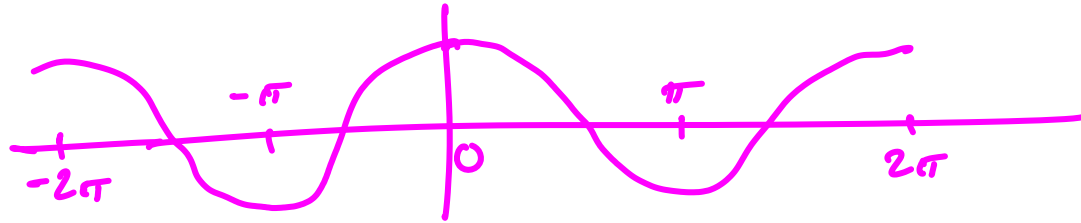
$$\int_1^3 \frac{dx}{x} = \left[\log x \right]_1^3 = \log 3 - \underbrace{\log 1}_{=0} = \log 3$$

$n \in \mathbb{Z}$

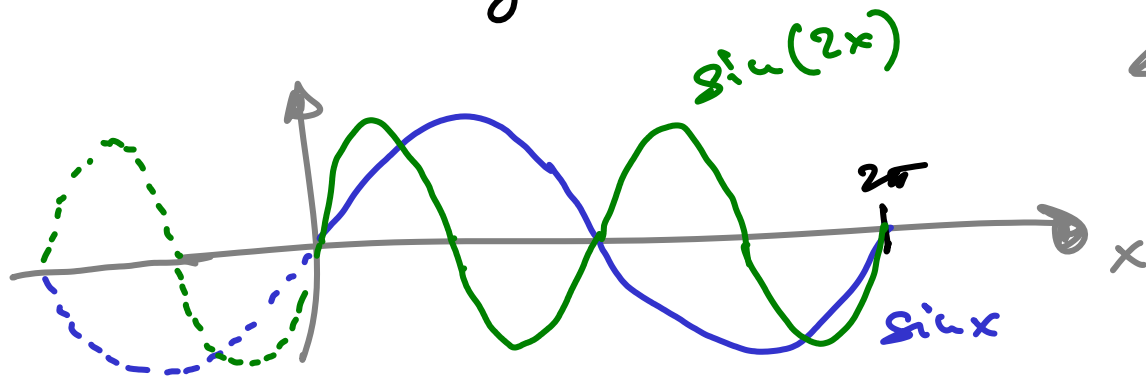
$$\int_{-\pi}^{\pi} \sin(nx) \, dx$$

$$= \left[-\cos(nx) \cdot \frac{1}{n} \right]_{-\pi}^{\pi} = -\frac{1}{n} \left[\cos(nx) \right]_{-\pi}^{\pi}$$

$$= -\frac{1}{n} \left(\cos(n\pi) - \cos(-n\pi) \right) = -\frac{1}{n} \left((-1)^n - (-1)^n \right) = 0$$



ohne Rechnung



Begründung für
 $\int_0^{2\pi} \sin(nx) \, dx = 0$

$$\int_{-\pi}^{\pi} \sin^2(ux) dx = \int_{-\pi}^{\pi} (\sin(ux))^2 dx$$

$$\cos(2x) = \cos^2 x - \sin^2 x \quad \text{Add. Thm. des cos}$$
$$= 1 - 2\sin^2 x$$

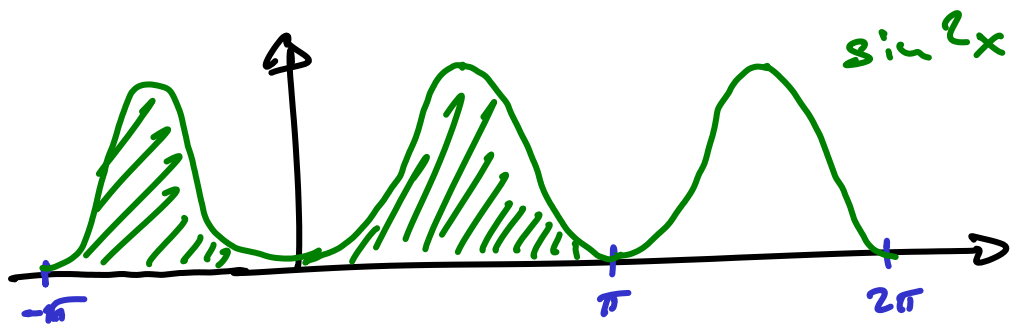
$$\text{Pyth. } \cos^2 x + \sin^2 x = 1$$

$$\Rightarrow \sin^2 x = \frac{1}{2} - \frac{1}{2} \cos(2x)$$

$$\int_{-\pi}^{\pi} \sin^2(ux) dx = \int_{-\pi}^{\pi} \left(\frac{1}{2} - \frac{1}{2} \cos(2ux) \right) dx$$

$$= \left[\frac{1}{2}x - \frac{1}{4u} \sin(2ux) \right]_{-\pi}^{\pi}$$

$$= \left(\frac{\pi}{2} - 0 \right) - \left(-\frac{\pi}{2} - 0 \right) = \pi$$



Produktregel: $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$

$$\int_a^b \dots dx \Rightarrow$$

$$\left[f(x)g(x) \right]_a^b = \int_a^b f'(x)g(x) dx + \int_a^b f(x)g'(x) dx$$

$$\Leftrightarrow \int_a^b f'(x)g(x) dx = \left[f(x)g(x) \right]_a^b - \int_a^b f(x)g'(x) dx$$

$$\int_0^{\pi/2} \underbrace{x}_{g} \cdot \underbrace{\cos x}_{g'} dx = \left[x \sin x \right]_0^{\pi/2} - \int_0^{\pi/2} 1 \cdot \sin x dx$$

$$= \frac{\pi}{2} \cdot 1 - 0 \cdot 0 - \left[-\cos x \right]_0^{\pi/2}$$

$$= \frac{\pi}{2} + \underbrace{\cos\left(\frac{\pi}{2}\right)}_{=0} - \underbrace{\cos(0)}_{=1} = \frac{\pi}{2} - 1$$

$$\int \underbrace{1}_{g'} \cdot \underbrace{\log x}_{g} dx = x \cdot \log x - \int \underbrace{x \cdot \frac{1}{x}}_{=1} dx = x \cdot \log x - x$$

Überprüfen:

$$\checkmark \text{ ist } (x \log x - x)' = \log x + x \cdot \frac{1}{x} - 1 = \log x \quad \text{😊}$$

$$n \neq m, \quad n, m \in \mathbb{N}_0$$

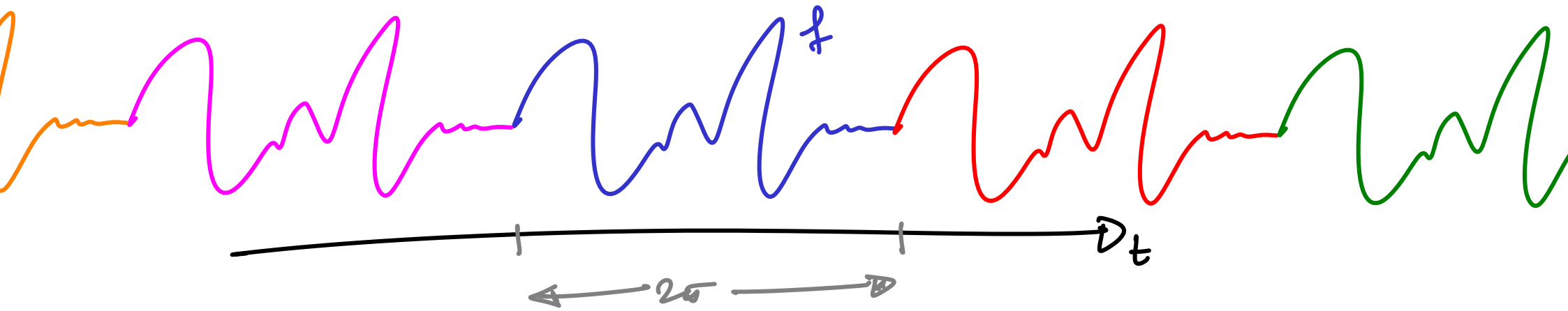
$$\int_{-\pi}^{\pi} \underbrace{\sin(nx)}_{f'} \underbrace{\sin(mx)}_g dx = \underbrace{\left[-\frac{\cos(nx)}{n} \sin(mx) \right]_{-\pi}^{\pi}}_{=0} + \int_{-\pi}^{\pi} \underbrace{\frac{\cos(nx)}{n}}_f \underbrace{\cos(mx)}_{g'} \cdot m dx$$

$$= \frac{m}{n} \int_{-\pi}^{\pi} \underbrace{\cos(nx)}_{f'} \underbrace{\cos(mx)}_g dx$$

$$= \frac{m}{n} \underbrace{\left[\frac{\sin(nx)}{n} \cos(mx) \right]_{-\pi}^{\pi}}_{=0} + \frac{m}{n} \int_{-\pi}^{\pi} \frac{\sin(nx)}{n} \sin(mx) \cdot m dx$$

$$= \frac{m^2}{n^2} \int_{-\pi}^{\pi} \sin(nx) \sin(mx) dx$$

$$\Rightarrow \underbrace{\left(1 - \frac{m^2}{n^2}\right)}_{\neq 0} \cdot \underbrace{\int_{-\pi}^{\pi} \sin(nx) \sin(mx) dx}_{=0} = 0$$



so eine Fkt. ist Summe von \sin - & \cos - Termen

$$\int_{-\pi}^{\pi} f(t) \sin(mt) dt = \underbrace{\int_{-\pi}^{\pi} a_0 \sin(mt) dt}_{=0} + \underbrace{\sum_n \int_{-\pi}^{\pi} a_n \cos(nt) \sin(mt) dt}_{=0 \text{ (ÜA)}} + \underbrace{\int_{-\pi}^{\pi} b_n \sin(nt) \sin(mt) dt}_{= \begin{cases} \pi b_n, & n=m \\ 0, & n \neq m \end{cases}}$$

$$= \pi b_m$$