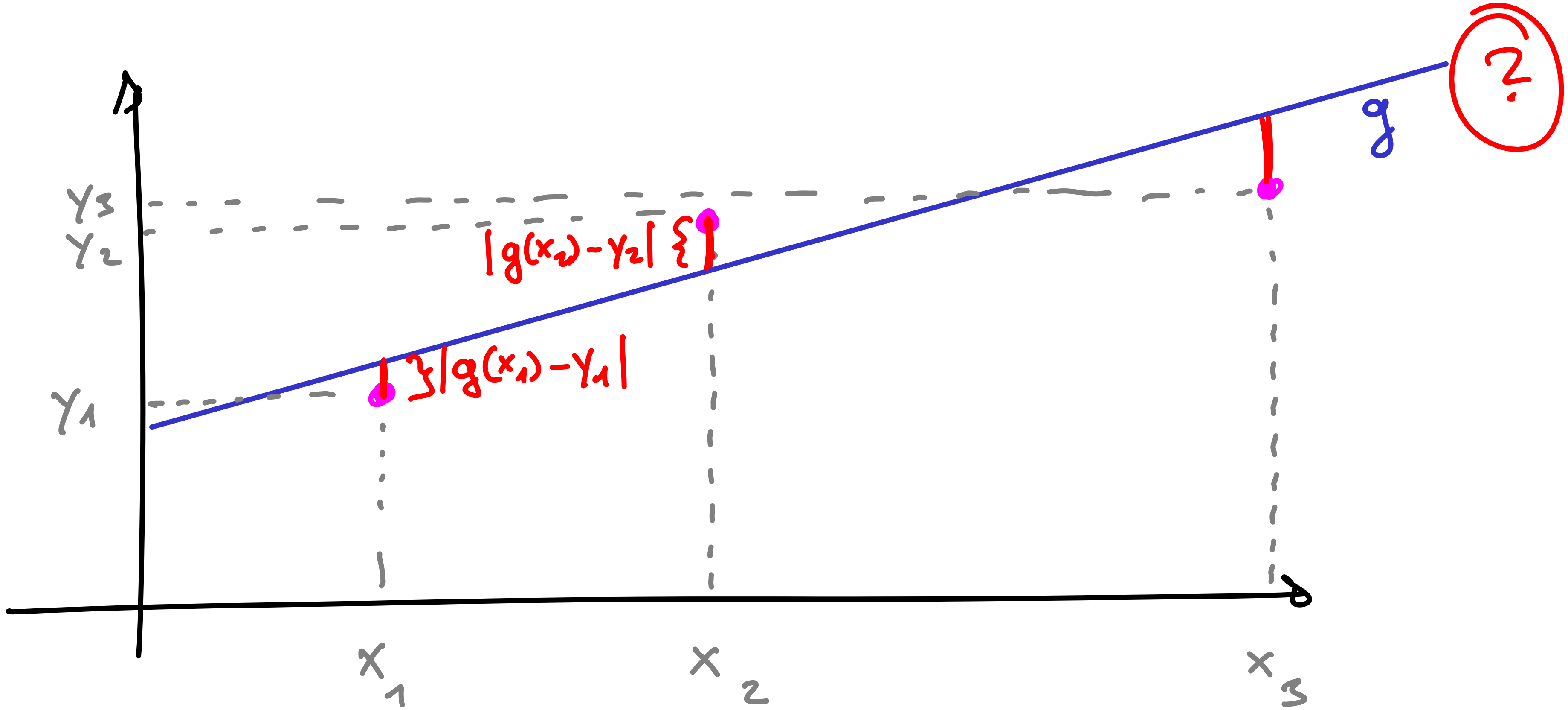

hier n=3

$$D(m,b)=\sqrt{\sum_{i=1}^{n}\left(g(x_i)-y_i\right)^2}$$

hier stecke m & b drin

$$=\sqrt{\sum_{i=1}^{n}\left(mx_i+b-y_i\right)^2}$$

$$f(m,b) = [D(m,b)]^2 = \sum_{i=1}^{n} (m x_i + b - y_i)^2$$

$$= (m x_1 + b - y_1)^2 + (m x_2 + b - y_2) + \dots$$

$$\frac{\partial f}{\partial b} = \sum_{i=1}^{n} 2(\underline{m} x_i + \underline{b} - y_i) \overset{!}{=} 0 \quad \Big| \cdot \frac{1}{2}$$

$$\frac{\partial f}{\partial m} = \sum_{i=1}^{n} 2(\underline{m} x_i + \underline{b} - y_i)\, x_i \overset{!}{=} 0 \quad \Big| \cdot \frac{1}{2}$$

$$\frac{\partial f}{\partial b} = 0 \iff n \cdot b + m \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$$

$$\frac{\partial f}{\partial m} = 0 \iff b \sum_{i=1}^{n} x_i + m \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i$$

$$\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})$$

$$= \sum_{i=1}^{n}\left(x_i y_i - x_i\bar{y} - \bar{x}y_i + \bar{x}\,\bar{y}\right)$$

$$= \sum_{i=1}^{n} x_i y_i - \bar{y}\underbrace{\sum_{i=1}^{n} x_i}_{=n\bar{x}} - \bar{x}\underbrace{\sum_{i=1}^{n} y_i}_{=n\bar{y}} + n\cdot\bar{x}\,\bar{y}$$

$$= \sum_{i=1}^{n} x_i y_i - n\,\bar{x}\,\bar{y}$$

mit $y_i \to x_i$ und $\sum_{i=1}^{n} x_i^2 - n\bar{x}^2 = \sum_{i=1}^{n}(x_i-\bar{x})^2$

Dürfte man durch

$$\sum_{i=1}^{n}\underbrace{(x_i-\bar{x})^2}_{\geq 0}$$ teilen ?

nur $=0$ falls alle x_i gleich also O.K.

Nochmal: Positive Definitheit von 2×2-Matrizen

$$A = \begin{pmatrix} a & b \\ b & d \end{pmatrix} \text{ symm. } 2\times 2\text{-Matrix}, \quad \vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\vec{x} \cdot (A\vec{x}) = \begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} ax+by \\ bx+dy \end{pmatrix} = ax^2 + bxy + byx + dy^2$$

$$= ax^2 + 2bxy + dy^2$$

$$= a\left(x^2 + \frac{2b}{a}xy\right) + dy^2$$

quadratische Ergänzung

$$= a\left(\left(x + \frac{b}{a}y\right)^2 - \frac{b^2}{a^2}y^2\right) + dy^2$$

$$= a\left(x + \frac{b}{a}y\right)^2 - \frac{b^2}{a}y^2 + dy^2$$

$$= a\underbrace{\left(x+\frac{b}{a}y\right)^2}_{\geq 0} + \frac{da-b^2}{a}\underbrace{y^2}_{\geq 0}$$

D.h. $A = \begin{pmatrix} a & b \\ b & d \end{pmatrix}$ ist positiv definit falls $a > 0$

<u>und</u> $ad - b^2 > 0$

Für $H = \begin{pmatrix} 2n & 2n\bar{x} \\ 2n\bar{x} & 2\sum_{i=1}^{n} x_i^2 \end{pmatrix}$

① $2n > 0$

② $$4n\sum_{i=1}^{n} x_i^2 - 4n^2\bar{x}^2 = 4n\underbrace{\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)}_{\substack{\text{Nenner von } m \\ \geq 0}}$$

d.h. es liegt tatsächlich ein Minimum vor

i	1	2	3	4	5	6
x_i	20	16	15	16	13	10
y_i	0	3	7	4	6	10

$\bar{x} = 15$, $\bar{y} = 5$

$$m = \frac{\sum_{i=1}^{6} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{6} (x_i - \bar{x})^2}$$

$$= \frac{(20-15)\cdot(0-5) + (16-15)\cdot(3-5) + (15-15)\cdot(7-5) + \dots + (10-15)(10-5)}{(20-15)^2 + (16-15)^2 + (15-15)^2 + \dots + (10-15)^2}$$