

## TRR 195 ANNUAL MEETING 2018, ABSTRACTS

### **Simon Brandhorst (Hannover), Finite groups of automorphisms of complex K3 surfaces**

A K3 surface is a compact complex surface which is simply connected and admits a global holomorphic symplectic form. Automorphisms leaving this form invariant are called symplectic. Finite groups acting symplectically on a K3 surface admit an intricate connection to the Mathieu group  $M_{23}$ . In this talk I survey this connection and report on work in progress to extend the classification beyond the symplectic case. Our approach is computer aided and utilizes Hermitian lattices. This is joint work with Kenji Hashimoto.

### **Nils Bruin (Simon Fraser University), Arithmetic applications of Prym varieties in low genus**

If a curve has a rational point, it can be embedded into its Jacobian variety. This is an abelian variety of dimension equal to the genus of the curve. This embedding gives a lot of information about the arithmetic properties of the curve, in particular about its rational points.

Under certain conditions, for instance if the curve admits an involution, there may be other abelian varieties available to embed the curve in. This leads to Prym varieties. In low genus, these Prym varieties are Jacobians themselves, but of different curves. Explicit, classical constructions lead to surprising relations between curves that can be used to obtain better computational methods for determining the rational points on curves. We will review some of these surprising correspondences and some ways to exploit them for arithmetic gain.

### **Alicia Dickenstein (University of Buenos Aires), Iterated discriminants and singular space curves**

In general, two quadric surfaces intersect in a nonsingular quartic space curve. If we relax the generality assumption, the intersection curve may degenerate to a finite number of different possible types of singular curves. The classification of such singular intersections goes back to T. J. IA. Bromwich (1906). L. Schläfli (1953) introduced conditions for a degenerate intersection of two surfaces of tensor type (or more generally, hypersurfaces described by multilinear equations).

Following ongoing joint work with S. di Rocco and R. Morrison, I will present a general framework of iterated discriminants to characterize the singular intersection of hypersurfaces with a given monomial support, which generalizes both previous situations. I will explain the notion of mixed discriminant and the relation with these iterated discriminants.

### **Christian Eder (TU Kaiserslautern), A Gröbner basis package for OSCAR**

In this talk we present the Gröbner basis library GB written in C which implements Faugere's F4 Algorithm. We explain the theory behind the implemented algorithms and variants, and give an overview over useful features added in the latest version. Moreover, we present the Julia wrapper GB.jl which integrates gb in the OSCAR computer algebra system and comes with a specific interface to read from and write to Singular. We conclude this talk with an outlook on new features and generalizations planned for the next versions.

### **Niamh Farrell (Kaiserslautern), Fake Galois actions**

Let  $G$  be a finite group of order  $n$ . The Galois group  $\text{Gal}(\mathbb{Q}_n/\mathbb{Q})$  permutes the set of ordinary irreducible characters of  $G$  but has, in general, no natural action on the set of Brauer characters of  $G$ . I will introduce the concept of fake Galois actions on Brauer characters, as defined by Spth and Vallejo. I will discuss the connection between fake Galois actions and a modular analogue of the

Glauberma Isaacs correspondence, and show how such actions can be constructed for the finite groups of Lie type of type A in non-defining characteristic.

**Ghislain Fourier (Hannover), Essential bases, semigroups and toric degenerations**

Let  $X$  be a Grassmann variety or (generalized) flag variety  $G/B$ . This talk will be mainly about finding an appropriate monomial basis of the homogeneous coordinate ring  $\mathbb{C}[X]$ . For this, I'll use representation theory of the algebraic group  $G$ , crystal and global bases, standard monomial theory and more recent developments such as birational sequences. There are various examples of well-behaved monomial bases, and much more cases, where such a basis is not yet found. How can GAP, Polymake and other programs be used to compute these bases? I'll close with several conjectures about the existence and properties of such monomial bases in general.

**Derek Holt (University of Warwick), Computing in finitely presented groups**

We survey algorithms for computing in (generally infinite) groups defined by finite presentations, with an emphasis on the existence and availability of effective implementations.

The talk will focus mainly on the Word Problem, the Generalised Word Problem and the Conjugacy Problem.

For the Word Problem we will concentrate on the use of automatic structures for defining an effective normal form for the groups elements, with some applications to deciding finiteness, and to drawing pictures.

There are not many methods available for the Generalised Word Problem when the index of the subgroup  $H$  of  $G$  is infinite. The Stallings Folding method for subgroups of free groups can be generalised to quasiconvex subgroups of automatic groups provided that we can compute a so-called coset automatic structure. This is possible, for example, for quasiconvex subgroups of hyperbolic groups.

The Conjugacy Problem in certain types of groups (braid groups and polycyclic groups, for example) has received a lot of attention recently resulting from potential applications to cryptography. But in this talk we focus on the problem in hyperbolic groups, which can theoretically be solved in (almost) linear time, but for which effective implementations seem to be more difficult.

**Michael Joswig (TU Berlin), The degree of a tropical basis**

We give an explicit upper bound for the degree of a tropical basis of a homogeneous polynomial ideal. As one application face vectors of tropical varieties are discussed. Various examples illustrate differences between Gröbner and tropical bases. Joint work with Benjamin Schröter.

**Yeongrak Kim (Saarland University), An explicit matrix factorization of cubic hypersurfaces of small dimension**

An early concept of matrix factorizations can be found in physics, especially by Dirac on the study of Heisenberg matrix mechanics. Eisenbud introduced this notion in mathematics to study homological algebra over hypersurface rings. Matrix factorizations become significant due to their various applications in commutative algebra, algebraic geometry, and mathematical physics. A particularly interesting case happens when a matrix factorization is induced by a linear matrix, since it provides a presentation of an Ulrich sheaf in such a case. The Ulrich complexity, the smallest possible rank of an Ulrich sheaf, is quite mysterious, and even is not exactly known for cubic hypersurfaces. On the other hand, Manivel recently shows that there are rank 9 Ulrich sheaves on a general cubic sevenfold using the invariant theory on the Lie group  $E_6$ . In this talk, I will give an alternative proof using Shamash's construction with the spinor variety. In particular, I will compute an explicit matrix factorization corresponds to a rank 9 Ulrich sheaf. Indeed, two arguments intersect on the geometry of the Cartan cubic. I will also give the complete classification of cubic forms whose Hessian induce matrix factorization of themselves. This is a joint work in a progress with F.-O. Schreyer.

**Robert Löwe (TU Berlin), Secondary fans and secondary polyhedra of punctured Riemann surfaces**

A famous construction of Gelfand, Kapranov, and Zelevinsky associates to each finite point configuration in  $\mathbb{R}^n$  its secondary fan, which stratifies the space of height functions by the combinatorial types of regular subdivisions. A completely analogous construction associates to each punctured Riemann surface a polyhedral fan, whose cones correspond to the ideal decompositions of the surface that occur as horocyclic Delaunay tessellations in the sense of Penner's convex hull construction. We call this fan the secondary fan of the punctured Riemann surface. Similar to the classical case, this secondary fan turns out to be the normal fan of a convex polyhedron, the secondary polyhedron of the Riemann surface. This is joint work with Michael Joswig and Boris Springborn.

**Frank Lübeck (RWTH Aachen), Computing with algebraic and finite reductive groups**

Except for cyclic, alternating and sporadic groups finite simple groups can be constructed via groups of fixed points  $G^F$  of a Steinberg morphism  $F$  on a simple connected reductive algebraic group  $G$ . Examples are  $G^F = SL_n(q) \subseteq SL_n(K) = G$ ,  $Spin_n^\pm(q) \subseteq Spin_n(K)$  or  $E_8(q) \subseteq E_8(K)$  where  $K$  is an algebraic closure of the field with  $q$  elements,  $q$  a power of a prime  $p$ .

In this talk I will sketch how these groups can be encoded in a simple data structure, called a root datum, and how this can be used for computer calculations. We are interested in collecting information on properties of elements, conjugacy classes, certain subgroups and the representations in various characteristics of these finite (and algebraic) groups.

I will sketch that a lot of information can be computed directly from the root datum (without a sometimes complicated construction of an explicit group), and how the results depend just on the root datum of the algebraic group, or on the characteristic of the ground field  $K$  or on the specific prime power  $q$ . In particular, I will introduce generic character tables which are parameterized character tables of an infinite series of groups.

**Gabriele Nebe (RWTH Aachen), Binary Hermitian lattices over number fields**

This is joint work with Markus Kirschmer. Using the arithmetic of quaternion algebras we developed an algorithm to classify totally positive definite quaternary quadratic lattices over totally real number fields. This can be applied to find representatives of isomorphism classes in certain genera of binary Hermitian lattices over totally complex number fields. In particular such lattices over cyclotomic fields have application to the classification of extremal lattices with certain automorphisms.

**Ion Nechita (Toulouse), On some applications of Weingarten calculus to quantum information theory**

We begin by reviewing Weingarten calculus, which is a way to evaluate averages of monomials in matrix entries over the unitary group. I will then briefly advertise a computer algebra package implementing this calculus symbolically, giving some examples. Finally, I will present some applications of Weingarten calculus to quantum information theory via some random matrix computations.

**Alice Niemeyer (Aachen), Algorithmically inspired probabilistic problems in group theory**

Groups play an important role in many areas of mathematics and science. Often it is possible to describe a group by a small generating set, yet computing information about a group can be very difficult. Modern algorithms answering questions about finite groups often rely on probabilistic methods. These in turn pose many interesting theoretical questions about groups, a few of which we will discuss in this talk.

**Marta Panizzut (TU Berlin), K3 Polytopes and their quartic surfaces**

The closure of the connected components of the complement of a tropical hypersurface are called regions. They have the structure of convex polyhedra. A 3-dimensional polytope is a K3 polytope if it is the closure of the bounded region of a smooth tropical quartic surface. In this talk we begin by studying properties of K3 polytopes. In particular we exploit their duality to regular unimodular central triangulations of reflexive polytopes in the fourth dilation of the standard tetrahedron. Then we focus on quartic surfaces that tropicalize to K3 polytopes, and we look at them through the lenses of Geometric Invariant Theory. We will highlight the computational aspect of this project. This is a joint work with Gabriele Balletti and Bernd Sturmfels.

**Bernd Sturmfels (MPI Leipzig), Varieties of Signature Tensors**

The signature of a parametric curve is a sequence of tensors whose entries are iterated integrals. This construction is central to the theory of rough paths in stochastic analysis. It is here examined through the lens of algebraic geometry. We introduce varieties of signature tensors for piecewise linear paths and on polynomial paths. These all live in universal varieties that are derived from free Lie algebras and their Lie groups. This is joint work with Carlos Amendola and Peter Friz.

**Oguzhan Yürük (TU Berlin), An Experimental Classification of Maximal Mediated Sets**

Maximal mediated sets (MMS), first introduced by Bruce Reznick, arise as a natural structure in the study of nonnegative polynomials supported on circuits. Due to Reznick's, de Wolff's, and Ilmanen's results, given a nonnegative polynomial  $f$  supported on a circuit  $C$  with vertex set  $\Delta$ ,  $f$  is a sum of squares if and only if the non-vertex element of  $C$  is in the MMS of  $\Delta$ . In this project, we classify MMS experimentally. As a main theoretical result, we show that an MMS is determined by its underlying lattice. This is joint work with Olivia Röhrig and Timo de Wolff.