FB Mathematik Winterterm 2004/05, Set 8
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# Algebraic Geometry I

Due date: Friday, 23/12/2005, 18:00 Uhr

**Exercise 1:** Let $X \subseteq \mathbb{A}^n_K$, $Y \subseteq \mathbb{A}^m_K$ be affine varieties, $f : X \to Y$ a morphism. Show that

$$I_{\mathbb{A}^m_K}(\overline{f(X)}) = I_{\mathbb{A}^{n+m}_K}(\Gamma_f) \cap K[y_1, \ldots, y_m],$$

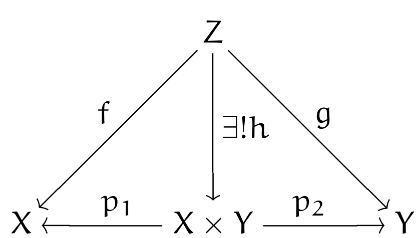
where $y_1, \ldots, y_m$ are coordinate functions of $\mathbb{A}^m_K$.

**Exercise 2:** Let $f : \mathbb{A}^2_\mathbb{C} \to \mathbb{A}^3_\mathbb{C}$ be a morphism given by

$$f : \mathbb{A}^2_\mathbb{C} \to \mathbb{A}^3_\mathbb{C}, \qquad (t, s) \mapsto (t, ts, s^2).$$

Compute the image of $f$ using SINGULAR and draw its real part with the help of the ray tracer `surf`.

**Exercise 3:** Let $X$, $Y$ be affine varieties, and let $p_1 : X \times Y \to X$ and $p_2 : X \times Y \to Y$ denote the canonical projections. Prove the following *universal property* of $X \times Y$: given morphisms of affine varieties $f : Z \to X$, $g : Z \to Y$, there exists a unique morphism $h : Z \to X \times Y$ such that $p_1 \circ h = f$, $p_2 \circ h = g$:




**Exercise 4:** Let $I$, $I_1$, $I_2$ be homogeneous ideals in $K[x_0, \ldots, x_n]$. Prove the following statements.

a. $\sqrt{I}$ is a homogeneous ideal.

b. $I_1 : I_2 = \{f \in K[x_0, \ldots, x_n] \mid f \cdot I_2 \subseteq I_1\}$ is a homogeneous ideal.

c. $I$ is a prime ideal if and only if for any *homogeneous* $f, g \in I$ holds:

$$f \cdot g \in I \Rightarrow f \in I \text{ or } g \in I.$$