FB Mathematik
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Winterterm 2005/06, Set 11
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Algebraic Geometry I

Problem 1: Let $C \subset \mathbb{P}^3$ be the twisted cubic curve, given as the image of the morphism $\nu_3 : \mathbb{P}^1 \to \mathbb{P}^3$, which sends $(t_0 : t_1)$ to $(t_0^3 : t_0^2 t_1 : t_0 t_1^2 : t_1^3)$. Show that C is not the complete intersection of two surfaces in $\mathbb{P}^3$, i.e. the ideal $I(C)$ is not generated by two elements. Moreover, describe the intersection of any two quadrics $Q_1 \neq Q_2$ in $\mathbb{P}^3$, both containing C.

Problem 2: Describe the blow-up of the following curves in $\mathbb{A}^2$:

(a) $x_1^{k+1} + x_2^2 \qquad k \geq 1$

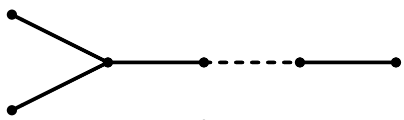
(b) $x_1^a + x_2^b \qquad \gcd(a, b) = 1$

Problem 3: Verify that the (minimal) desingularisation of each of the following surface singularities has an exceptional locus, all components of which are isomorphic to $\mathbb{P}^1$. These components intersect as indicated by the dual graph.

A_n: $x_0^{n+1} + x_1^2 + x_2^2$

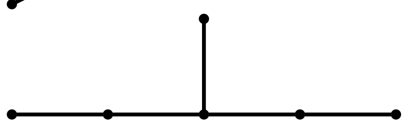
$n \geq 1$ vertices

D_n: $x_0^{n-1} + x_0 x_1^2 + x_2^2$

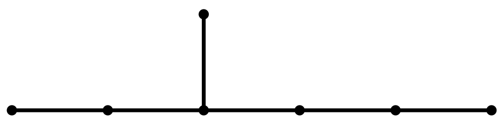


$n \geq 4$ vertices

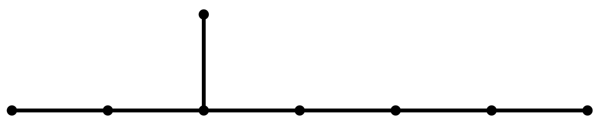
E_6: $x_0^4 + x_1^3 + x_2^2$



E_7 : $x_0^3 x_1 + x_1^3 + x_2^2$



E_8: $x_0^5 + x_1^3 + x_2^2$



Problem 4: For each integer $0 \leq k \leq 16$ find a quartic surface $S \subset \mathbb{P}^3$ with precisely k ordinary double points and no other singularities.