FB Mathematik — Winterterm 2005/06, Set 13
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# Algebraic Geometry I

Due date: Friday, 10/02/2006, 18:00 Uhr

**Exercise 1:** Let $(X, \mathcal{O}_X)$ be an affine variety and $(Y, \mathcal{O}_Y)$ a prevariety. Prove that the map

$$\mathrm{Mor}(Y, X) \to \mathrm{Hom}_{K-\mathrm{Alg}}(\mathcal{O}_X(X), \mathcal{O}_Y(Y)), \qquad (f : Y \to X) \mapsto (f^* : \varphi \mapsto f^*\varphi = \varphi \circ f)$$

is bijective.

**Exercise 2:** Let $(X, \mathcal{O}_X)$, $(Y, \mathcal{O}_Y)$ be ringed spaces. Define

$$\pi_X : X \times Y \to X, \qquad (x, y) \mapsto x,$$
$$\pi_Y : X \times Y \to Y, \qquad (x, y) \mapsto y.$$

Prove that

a. $\pi_X$ and $\pi_Y$ are morphisms of ringed spaces;

b. $\pi_X$ and $\pi_Y$ are open maps (i.e., map open sets to open sets).

**Exercise 3:** Prove that $\mathbb{A}^m \times \mathbb{A}^n \cong \mathbb{A}^{m+n}$.

*Hint.* Use the universal property of the product and Exercise 1.

**Exercise 4:** Let $(X, \mathcal{O}_X)$, $(Y, \mathcal{O}_Y)$ be ringed spaces and let $X_1$ resp. $Y_1$ be locally closed (open, closed) subspaces of $X$ resp. $Y$. Then $X_1 \times Y_1$ is a locally closed (open, closed) subspace of $X \times Y$ (as a ringed space).