Fachbereich Mathematik
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Summer Semester 2004, Set 1

# Computer Algebra

Due date: Monday, 10/05/2004, 10h00

**Exercise 1:** Let $>$ be any monomial ordering on $\text{Mon}_n$.

a. Show that for a fixed $w = (w_1, \ldots, w_n) \in \mathbb{R}^n$ the following defines a monomial ordering on $\text{Mon}_n$:

$$\underline{x}^\alpha >_{(w,>)} \underline{x}^\beta \;:\Longleftrightarrow\; \langle w, \alpha\rangle > \langle w, \beta\rangle, \;\text{ or }\; \big(\langle w, \alpha\rangle = \langle w, \beta\rangle \;\text{ and }\; \underline{x}^\alpha > \underline{x}^\beta\big).$$

Under which assumptions is the above definition independent of the chosen ordering $>$?

b. Let $A \in \text{Gl}_n(\mathbb{Q})$ be an invertible $n \times n$-matrix over the rational numbers. Show that the following defines a monomial ordering on $\text{Mon}_n$:

$$\underline{x}^\alpha >_{(A,>)} \underline{x}^\beta \;:\Longleftrightarrow\; \underline{x}^{A\alpha} > \underline{x}^{A\beta}.$$

**Exercise 2:** Determine matrices $A \in \text{Gl}_n(\mathbb{R})$ which define the following orderings:

a. $(>_{lp}, >_{ds})$, here $n = n_1 + n_2$ with $n_1$ variables for $lp$ and $n_2$ for $ds$;

b. $(>_{dp}, >_{ls})$, here $n = n_1 + n_2$ with $n_1$ variables for $dp$ and $n_2$ for $ls$;

c. $>_{wp(5,3,4)}$, here $n = 3$;

d. $>_{ws(5,3,4)}$, here $n = 3$.

**Exercise 3:** Let $M \subset \text{Mon}_n$ be a finite subset and $> = >_{lp}$ the lexicographical ordering. Construct a weight vector $w$ which defines $>$ on $M$, that is such that for $\underline{x}^\alpha, \underline{x}^\beta \in M$:

$$\big(\underline{x}^\alpha > \underline{x}^\beta \;\Longleftrightarrow\; \langle w, \alpha\rangle > \langle w, \beta\rangle\big).$$