Dr. Thomas Keilen

Computer Algebra

Due date: Monday, 21/06/2004, 10h00

Exercise 17: Let $I \trianglelefteq K[\underline{x}]$ and $>$ a monomial ordering on $K[\underline{x}]$. Show there is an $\omega \in (\mathbb{Z} \setminus \{0\})^n$ such that $L_>(I) = L_{>_{(\omega,>_{lp})}}(I)$.

Exercise 18: Let R be a ring, $I \trianglelefteq R$, $g_0, \ldots, g_l \in R$ and $g = \sum_{i=0}^{l} g_i t^i \in R[t]$. Show that $I : \langle g_0, \ldots, g_l \rangle = \big(I \cdot R[t] : \langle g \rangle\big) \cap R$.

Exercise 19: Check (by hand) whether $f = xz^3 - 2y^2$ belongs to the ideal $I = \big\langle xy - y, 2x^2 + yz, y - z \big\rangle_R$ for

a. $R = \mathbb{Q}[x, y, z]$, respectively

b. $R = \mathbb{Q}[x, y, z]_{\langle x,y,z \rangle}$.

Exercise 20: Write a SINGULAR procedure `standardbasis` which takes as input a list consisting of polynomials $f_1, \ldots, f_k$ and returns a standard basis of the ideal generated by $f_1, \ldots, f_k$.

Remark: Use the polynomial normal form PNFMora and build in the product criterion in order to speed up the computations.