Fachbereich Mathematik
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Summer Semester 2004, Set 6

Computer Algebra

Due date: Monday, 28/06/2004, 10h00

Exercise 21: Let $I \trianglelefteq K[\underline{x}]$ be an ideal, $>$ a global monomial ordering, and $B = \text{Mon}(\underline{x}) \cap \big(K[\underline{x}] \setminus L(I)\big)$ the set of monomials which are not in the leading ideal of I. Show that B is a K-vector space basis of $K[\underline{x}]/I$.

Exercise 22: [Computation in $K[\underline{x}]/I$ via Normal Forms]

Let $>$ be a global monomial ordering on $K[\underline{x}]$ and NF be a *reduced* normal form on $K[\underline{x}]$ w. r. t. $>$. Show:

a. $\text{NF}(g, G) = \text{NF}(g, G')$ for all $g \in K[\underline{x}]$ and for all standard bases G and G' of I.

   We therefore may define $\text{NF}(g, I) := \text{NF}(g, G)$.

b. $\text{NF}(g, I) + \text{NF}(g', I) = \text{NF}(g + g', I)$ for all $g, g' \in K[\underline{x}]$.

c. $\text{NF}\big(\text{NF}(g, I) + \text{NF}(g', I), I\big) = \text{NF}(g \cdot g', I)$ for all $g, g' \in K[\underline{x}]$.

Exercise 23:

a. Change your procedure `standardbasis` in such a way that it takes an optional parameter. If the optional paramerter is the `string` *“minimal”* it returns a minimal standard basis, if the optional parameter is the `string` *“reduced”* it returns a reduced standard basis, and if the optional parameter is missing, it just returns some standard basis as before.

b. Write a SINGULAR procedure `radicalmemebership` which takes as input a polynomial g and a list of polynomials $f_1, \ldots, f_k$, and which returns 1 if $g \in \sqrt{\langle f_1, \ldots, f_k \rangle}$, and 0 else.

Hint, if you define the head of the procedure `standardbasis` as `proc standardbasis (ideal G, list #)`, then `#` is an optional parameter of type list and with `size(#)==0` you can test whether it is there or not, while with #[1] you can access its entry if it is there.

Exercise 24:

a. Write a SINGULAR procedure `intersection` which takes as input two lists consisting of polynomials $f_1, \ldots, f_k$ respectively $g_1, \ldots, g_l$ and returns a standard basis of the ideal $\langle f_1, \ldots, f_k \rangle \cap \langle g_1, \ldots, g_l \rangle$.

b. Write a SINGULAR procedure `idealquotient` which takes as input two lists consisting of polynomials $f_1, \ldots, f_k$ respectively $g_1, \ldots, g_l$ and returns a standard basis of the ideal $\langle f_1, \ldots, f_k \rangle : \langle g_1, \ldots, g_l \rangle$.