Fachbereich Mathematik
Dr. Thomas Keilen

Summer Semester 2004, Set 7

Computer Algebra

Due date: Monday, 05/07/2004, 10h00

Exercise 25: Let $>= (c, >_{dp})$ on $\text{Mon}^2(x, y)$ and $G = \big((x^2, xy)^t, (x, y^2)^t\big)$. Compute the reduced normal form of $g = (x^2 + y^2 + 2x, y - 1)^t$ with respect to G.

Exercise 26: Let $>$ be a global monomial ordering on $\text{Mon}(\underline{x})$, and let $M \in \text{Mat}(n \times n, K[\underline{x}])$. By $f_1, \ldots, f_n \in K[\underline{x}]^{2n}$ we denote the rows of the matrix $(M, \mathbb{1}_n)$, and $G = (g_1, \ldots, g_k)$ shall be *the* reduced standard basis of $\langle g_1, \ldots, g_k \rangle_{K[\underline{x}]} \leq K[\underline{x}]^{2n}$ w. r. t. the ordering $(c, >)$ with $\text{lm}(g_1) > \ldots > \text{lm}(g_k)$. Show:

a. M is invertible if and only if $k = n$ and $\text{lm}(g_i) = e_i$ for $i = 1, \ldots, n$.

b. If M is invertible, then the rows of $(\mathbb{1}_n, M^{-1})$ are just $g_1, \ldots, g_n$.

Exercise 27: Let $f, g \in K[\underline{x}]$. Express $\gcd(f, g)$ and $\text{lcm}(f, g)$ in terms of elements in $\text{syz}(f, g)$ and derive an algorithm to compute these, assuming we can compute a standard basis of $\text{syz}(f, g)$.

Exercise 28: Write a SINGULAR procedure `chaincriterion` which takes a list of pairs of polynomials as input and eliminates pairs using the chain criterion. Then adjust your procedure `standardbasis` with this procedure.