Fachbereich Mathematik
Dr. Thomas Keilen

Summer Semester 2004, Set 8

Computer Algebra

Due date: Wednesday, 22/07/2004, 10h00

Exercise 29: Let $R = \mathbb{Q}[x,y,z]/\langle x^2+y^2+z^2\rangle$, $M = R^3/\langle (x,xy,xz)^t\rangle$ and $N = R^2/\langle (1,y)^t\rangle$. Moreover, let $\varphi : M \to N$ be given by the matrix

$$A = \begin{pmatrix} x^2+1 & y & z \\ yz & 1 & -y \end{pmatrix}.$$

a. Compute $\mathrm{Ker}(\varphi)$.

b. Test if $(x^2,y^2)^t \in \mathrm{Im}(\varphi)$.

c. Compute $\mathrm{Im}(\varphi) \cap \{f \in N \mid f \equiv (h,0) \bmod \langle (x,1)^t\rangle \text{ for some } h \in R\}$.

d. Compute $\mathrm{ann}_R\big(\mathrm{Im}(\varphi)\big)$.

Note, you may use Singular for your computations!

Exercise 30: Write a SINGULAR procedure `noethernormalisation` which takes as input an ideal I in the polynomial ring $K[\underline{x}]$ and returns a list (M,d) such that $K[x_1,\ldots,x_d] \hookrightarrow K[\underline{x}]/\Phi_{M^{-1}}(I)$ is a Noether normalisation.