Fachbereich Mathematik
Dr. Thomas Keilen

Summer Semester 2004, Set 9

Computer Algebra

Due date: Wednesday, 29/07/2004, 10h00

Exercise 31: Let R be a ring, $I \trianglelefteq R$, and $f, g \in R$ such that $\langle f, g\rangle = R$ and $f \cdot g \in I$. Then $I = \langle I, f\rangle \cap \langle I, g\rangle$.

Exercise 32: Let $\overline{K}$ the algebraic closure of K, and $I \trianglelefteq K[\underline{x}]$. Show, $I \cdot \overline{K}[\underline{x}] \cap K[\underline{x}] = I$.

Exercise 33: Let K be a field with $\mathrm{char}(K) = 0$ and let $I \trianglelefteq K[\underline{x}]$ with $\dim(K[\underline{x}]/I) = 0$. Show, if $\sqrt{I \cap K[x_i]} = \langle f_i\rangle$, then $\sqrt{I} = I + \langle f_1, \ldots, f_n\rangle$.

Hint, consider a primary decomposition of $(I\langle f_1, \ldots, f_n\rangle) \cdot \overline{K}[\underline{x}]$ induced by factorizing each f_i into linear factors over $\overline{K}$ and applying Exercise 31.

Exercise 34:

a. Deduce from Exercise 33 an algorithm `ZDRadical` which computes the radical of a zero-dimensional ideal in $K[\underline{x}]$ with $\mathrm{char}(K) = 0$. You may assume that we can calculate the squarefree part of a univariate polynomial (e. g. by factorizing it).

b. Deduce an algorithm `Radical` for an arbitrary ideal $I \trianglelefteq K[\underline{x}]$ where $\mathrm{char}(K) = 0$ via reduction to dimension zero as for the primary decomposition.

Remark, for b. you need Proposition 9.20 and Algorithm 9.21 from the lecture.