Fachbereich Mathematik
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Summer Semester 2006, Set 2

Computer Algebra

Exercise 5: Let R be a unique factorisation domain and $S \subset R$ a multiplicatively closed subset. Show that $S^{-1}R$ is a unique factorisation domain.

Hint, use the one-to-one correspondance of prime ideals under localisation.

Exercise 6: Let $>$ be a local ordering on $\mathrm{Mon}(x_1, \ldots, x_n)$. Show that

$$K(y_1, \ldots, y_m)[x_1, \ldots, x_n]_> = K[x_1, \ldots, x_n, y_1, \ldots, y_m]_{\langle x_1, \ldots, x_n \rangle}.$$

Exercise 7: Give one possible realization of the following rings within SINGULAR:

a. $\mathbb{Q}[x, y, z]$,

b. $\mathbb{F}_5[x, y, z]$,

c. $\mathbb{Q}[x, y, z]/\langle x^5 + y^3 + z^2 \rangle$,

d. $\mathbb{Q}(i)[x, y]$, where i is the imaginary unit,

e. $\mathbb{F}_{27}[x_1, \ldots, x_{10}]_{\langle x_1, \ldots, x_{10} \rangle}$,

f. $\mathbb{F}_{32003}[x, y, z]_{\langle x,y,z \rangle}/\langle x^5 + y^3 + z^2, xy \rangle$,

g. $\mathbb{Q}(t)[x, y, z]$,

h. $\big(\mathbb{Q}[t]/\langle t^3 + t^2 + 1 \rangle\big)[x, y, z]_{\langle x,y,z \rangle}$,

i. $\mathbb{Q}[x, y, z]_{\langle x,y \rangle}$.

Exercise 8: Write a SINGULAR procedure `spolynomial` which takes as input two polynomials and returns their s-polynomial.