Fachbereich Mathematik
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Computer Algebra

Exercise 9: Let $I \trianglelefteq K[\underline{x}]$ be a homogeneous ideal in the polynomial ring. Show that the degree reverse lexicographical ordering $>_{dp}$ satisfies the properties:

$$L\big(I + x_n^d\big) = L(I) + \langle x_n^d \rangle \qquad \text{and} \qquad L\big(I : x_n^d\big) = L(I) : x_n^d \qquad \text{for any } d \geq 1.$$

Note, since I is homogeneous we only have to consider homogeneous polynomials – e. g. $L(I) = \langle \mathrm{lm}(f) \mid f \in I, f \text{ homogeneous} \rangle$, etc.

Exercise 10: Apply IDBUCHBERGER to the following data (without using `Singular`):

$$g = x^4 + y^4 + z^4 + xyz, G = \left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z} \right), > = >_{dp} .$$

Exercise 11: Write a SINGULAR procedure `IDBuchberger` which takes as input a list consisting of a polynomial g and a list of polynomials $f_1, \ldots, f_k$ and returns a standard representation of g with respect to $(f_1, \ldots, f_k)$ in the form of a list consisting of the remainder r and a list of the scalars $q_1, \ldots, q_k$.

Note, the ordering of the base ring must be global! For a list of polynomials it is best to use the type `ideal`.

Exercise 12: Write a SINGULAR procedure `RedIDBuchberger` which takes as input a list consisting of a polynomial g and a list of polynomials $f_1, \ldots, f_k$ and returns a reduced standard representation of g with respect to $(f_1, \ldots, f_k)$ in the form of a list consisting of the remainder r and a list of the scalars $q_1, \ldots, q_k$. I suggest that you do your implementation in such a way that you actually get a determinate division with remainder.

Note, the ordering of the base ring must be global!