

Computer Algebra

Exercise 13: Let $\succ = (c, \succ_{dp})$ on $\text{Mon}^2(x, y)$ and $G = ((x^2, xy)^t, (x, y^2)^t)$. Compute the determinate division with remainder of $g = (x^2 + y^2 + 2x, y - 1)^t$ with respect to G (without using SINGULAR).

Exercise 14: Let \succ be any monomial ordering on Mon_n , and let $f, g \in K[\underline{x}]$ with $\gcd(\text{lm}(f), \text{lm}(g)) = 1$.

- Show there is a polynomial division with remainder of $\text{spoly}(f, g)$ with respect to (f, g) with remainder zero.
- Show that for each G containing f and g there is a polynomial division with remainder of $\text{spoly}(f, g)$ with respect G whose remainder is zero.

Hint, show first that $\text{spoly}(f, g) = a_0 f + b_0 g$ for $a_0 = -\text{tail}(g)$ and $b_0 = \text{tail}(f)$, and then define recursively $a_i = \text{tail}(a_{i-1})$ and $b_i = \text{tail}(b_{i-1})$. Consider the maximal value N such that $u \cdot \text{spoly}(f, g) = a_N f + b_N g$ for some unit $u \in K[\underline{x}]^* \cap K[\underline{x}]$, and distinguish the two cases that $\text{lt}(a_N f) + \text{lt}(b_N g)$ vanishes respectively does not vanish.

Exercise 15: Write a SINGULAR procedure `PIDMora` which takes as input a list consisting of a polynomial g and a list of polynomials f_1, \dots, f_k and returns a polynomial division with remainder of g with respect to (f_1, \dots, f_k) , i. e. a list consisting of a unit u , a list of scalars (q_1, \dots, q_k) , and a remainder r such that $u \cdot g = \sum_{i=1}^k q_i \cdot f_i + r$ is a PDwR.

Exercise 16: Write a SINGULAR procedure `standardbasis` which takes as input a list consisting of polynomials f_1, \dots, f_k and returns a standard basis of the ideal generated by f_1, \dots, f_k .

Remark: Use the polynomial division with remainder `PIDMora` and build in the product criterion in order to speed up the computations.