

Computer Algebra

Exercise 17: Let R be a ring, $I \trianglelefteq R$, $g_0, \dots, g_l \in R$ and $g = \sum_{i=0}^l g_i t^i \in R[t]$. Show that $I : \langle g_0, \dots, g_l \rangle = (I \cdot R[t] : \langle g \rangle) \cap R$.

Exercise 18: Check (by hand) whether $f = xz^3 - 2y^2$ belongs to the ideal $I = \langle xy - y, 2x^2 + yz, y - z \rangle_R$ for

a. $R = \mathbb{Q}[x, y, z]$, respectively

b. $R = \mathbb{Q}[x, y, z]_{\langle x, y, z \rangle}$.

Exercise 19: Change your procedure `standardbasis` in such a way that it takes an optional parameter. If the optional parameter is the string *“minimal”* it returns a minimal standard basis, if the optional parameter is the string *“reduced”* it returns a reduced standard basis, and if the optional parameter is missing, it just returns some standard basis as before.

Hint, if you define the head of the procedure `standardbasis` as `proc standardbasis (ideal G, list #)`, then `#` is an optional parameter of type list and with `size(#)==0` you can test whether it is there or not, while with `#[1]` you can access its entry if it is there.

Exercise 20: Write a SINGULAR procedure `radicalmembership` which takes as input a polynomial g and a list of polynomials f_1, \dots, f_k , and which returns 1 if $g \in \sqrt{\langle f_1, \dots, f_k \rangle}$, and 0 else.