Fachbereich Mathematik Summer Semester 2006, Set 6

Dr. Thomas Markwig

Computer Algebra

Exercise 21: Let $I \trianglelefteq K[\underline{x}]$ be an ideal, $>$ a global monomial ordering, and $B = \operatorname{Mon}(\underline{x}) \cap \big(K[\underline{x}] \setminus L(I)\big)$ the set of monomials which are not in the leading ideal of I. Show that B is a K-vector space basis of $K[\underline{x}]/I$.

Exercise 22: [Computation in $K[\underline{x}]/I$ via Normal Forms]

Let $>$ be a *global* monomial ordering on Mon_n^m, $g \in K[\underline{x}]^m$ and $I \leq K[\underline{x}]^m$.

a. If G and G' are two standard bases of I w.r.t. $>$ and $r \in K[\underline{x}]^m$ respectively $r' \in K[\underline{x}]^m$ is the remainder of a *reduced* standard representation of g w.r.t. G respectively G', then $r = r'$.

   We may therefore call $\operatorname{NF}(g, I) := r$ the *normal form* of g w.r.t. I and $>$.

b. $\operatorname{NF}(g, I) + \operatorname{NF}(g', I) = \operatorname{NF}(g + g', I)$ for all $g, g' \in K[\underline{x}]$.

c. $\operatorname{NF}\big(\operatorname{NF}(g, I) \cdot \operatorname{NF}(g', I), I\big) = \operatorname{NF}(g \cdot g', I)$ for all $g, g' \in K[\underline{x}]$.

Exercise 23: Write a SINGULAR procedure `intersection` which takes as input two lists consisting of polynomials $f_1, \ldots, f_k$ respectively $g_1, \ldots, g_l$ and returns a standard basis of the ideal $\langle f_1, \ldots, f_k\rangle \cap \langle g_1, \ldots, g_l\rangle$.

Exercise 24: Write a SINGULAR procedure `idealquotient` which takes as input two lists consisting of polynomials $f_1, \ldots, f_k$ respectively $g_1, \ldots, g_l$ and returns a standard basis of the ideal $\langle f_1, \ldots, f_k\rangle : \langle g_1, \ldots, g_l\rangle$.