Fachbereich Mathematik
Dr. Thomas Markwig Summer Semester 2006, Set 8

Computer Algebra

Exercise 29: Write a SINGULAR procedure `noethernormalisation` which takes as input an ideal I in the polynomial ring $K[\underline{x}]$ and returns a list (M, d) such that $K[x_1, \ldots, x_d] \hookrightarrow K[\underline{x}]/\Phi_{M^{-1}}(I)$ is a Noether normalisation.

Exercise 30: Let R be a ring, $I \trianglelefteq R$, and $f, g \in R$ such that $\langle f, g\rangle = R$ and $f \cdot g \in I$. Then $I = \langle I, f\rangle \cap \langle I, g\rangle$.

Exercise 31: Let $\overline{K}$ the algebraic closure of K, and $I \trianglelefteq K[\underline{x}]$. Show, $I \cdot \overline{K}[\underline{x}] \cap K[\underline{x}] = I$.

Exercise 32: Let K be a field with $\text{char}(K) = 0$ and let $I \trianglelefteq K[\underline{x}]$ with $\dim(K[\underline{x}]/I) = 0$. Show, if $\sqrt{I \cap K[x_i]} = \langle f_i\rangle$, then $\sqrt{I} = I + \langle f_1, \ldots, f_n\rangle$.

Hint, consider a primary decomposition of $(I\langle f_1, \ldots, f_n\rangle) \cdot \overline{K}[\underline{x}]$ induced by factorizing each f_i into linear factors over $\overline{K}$ and applying Exercise 31.

Exercise 33:

a. Deduce from Exercise 33 an algorithm `ZDRadical` which computes the radical of a zero-dimensional ideal in $K[\underline{x}]$ with $\text{char}(K) = 0$. You may assume that we can calculate the squarefree part of a univariate polynomial (e. g. by factorizing it).

b. Deduce an algorithm `Radical` for an arbitrary ideal $I \trianglelefteq K[\underline{x}]$ where $\text{char}(K) = 0$ via reduction to dimension zero as for the primary decomposition.

Remark, for b. you need Proposition 7.20 and Algorithm 7.21 from the lecture, so this cannot be done until next week.