Fachbereich Mathematik
Dr. Thomas Keilen

Winter Semester 2004/05, Set 5

Commutative Algebra

Due date: Monday, 10/01/2005, 14h00

Exercise 16: Let R be a UFD and $f \in R[x]$ be irreducible. Show that f is prime.

Hint, for $\deg(f) > 0$ pass to $K = \mathrm{Quot}(R)$ and use 3.14.

Exercise 17: Let R be a ring, $f \in R$ a non-zero-divisor. Show $R_f \cong R[x]/\langle fx - 1\rangle$.

Exercise 18: Let R be a ring and $\mathcal{N}(R)$ its nilradical. Show:

a. If $S \subseteq R$ multiplicatively closed, then $\mathcal{N}(S^{-1}R) = S^{-1}\mathcal{N}(R)$.

b. A ring is called *reduced* if it has no nilpotent elements except 0. Show that "being reduced" is a local property.

Exercise 19: Let K be a field, $R = K[x, y, z]/\langle xz, yz\rangle$ and $P = \langle x, y, z - 1\rangle \trianglelefteq R$. Show $R_P \cong K[z]_{\langle z-1\rangle}$.