Fachbereich Mathematik — Winter Semester 2021/22, Set 14

Thomas Markwig — Alheydis Geiger

Commutative Algebra

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Exercise 43: Compute the dimension of $K[x,y,z]_{\langle x^2-yz\rangle}$.

Exercise 44: Prove the Algebraic HNS (Theorem 7.1) using Noether-Normalisation.

Exercise 45: Let R be a ring. Show that $\dim(R[x]) \geq \dim(R)+1$.

Hint, consider ideals of the form $I[x] = \left\{ \sum_{i=0}^{n} a_i x^i \mid n \geq 0, a_i \in I \right\}$ for some ideal $I \trianglelefteq R$. – Note, if R is noetherian one can actually show equality, but that is much harder.

Exercise 46: Compute $\mathrm{Quot}(R)$ and $\dim(\mathrm{Quot}(R))$ for $R = K[x,y]/\langle x^2, xy\rangle$.

In class exercise 29: Let R be a finitely generated K-algebra which is an integral domain and let $K' = \mathrm{Quot}(R)$. Show that:

a. If $\beta_1,\ldots,\beta_d \in R$ are algebraically independent over K and R is algebraic over $K[\beta_1,\ldots,\beta_d]$, then $\mathrm{Quot}(R)$ is algebraic over $K(\beta_1,\ldots,\beta_d)$.

b. $\mathrm{trdeg}_K(R) = \mathrm{trdeg}_K(K')$.