Fachbereich Mathematik
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Winter Semester 2007/08, Set 4

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Computer Algebra

Due date: Tuesday, 27/11/2007, 10h00

Exercise 13: Let $> = (c, >_{dp})$ on $\text{Mon}^2(x, y)$ and $G = \big((x^2, xy)^t, (x, y^2)^t\big)$. Compute the determinate division with remainder of $g = (x^2 + y^2 + 2x, y - 1)^t$ with respect to G (without using SINGULAR).

Exercise 14: Let $>$ be any monomial ordering on Mon_n, and let $f, g \in K[\underline{x}]$ with $\gcd\big(\text{lm}(f), \text{lm}(g)\big) = 1$.

a. Show there is a polynomial division with remainder of $\text{spoly}(f, g)$ with respect to (f, g) with remainder zero.

b. Show that for each G containing f and g there is a polynomial division with remainder of $\text{spoly}(f, g)$ with respect G whose remainder is zero.

Hint, show first that $\text{spoly}(f, g) = a_0 f + b_0 g$ for $a_0 = -\text{tail}(g)$ and $b_0 = \text{tail}(f)$, and then define recursively $a_i = \text{tail}(a_{i-1})$ and $b_i = \text{tail}(b_{i-1})$. Consider the maximal value N such that $u \cdot \text{spoly}(f, g) = a_N f + b_N g$ for some unit $u \in K[\underline{x}]^*_> \cap K[\underline{x}]$, and distinguish the two cases that $\text{lt}(a_N f) + \text{lt}(b_N g)$ vanishes respectively does not vanish.

Exercise 15: Let R be a ring, $I \trianglelefteq R$, $g_0, \ldots, g_l \in R$ and $g = \sum_{i=0}^{l} g_i t^i \in R[t]$. Show that $I : \langle g_0, \ldots, g_l \rangle = \big(I \cdot R[t] : \langle g \rangle\big) \cap R$.

Exercise 16: Write a SINGULAR procedure `PIDMora` which takes as input a list consisting of a polynomial g and a list of polynomials $f_1, \ldots, f_k$ and returns a polynomial division with remainder of g with respect to $(f_1, \ldots, f_k)$, i. e. a list consiting of a unit u, a list of scalars $(q_1, \ldots, q_k)$, and a remainder r such that $u \cdot g = \sum_{i=1}^{k} q_i \cdot f_i + r$ is a PDwR.