Fachbereich Mathematik
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Winter Semester 2007/08, Set 6

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Computer Algebra

Due date: Tuesday, 11/12/2007, 10h00

Exercise 21: Let $I \trianglelefteq K[\underline{x}]$ be an ideal, $>$ a global monomial ordering, and $B = \mathrm{Mon}(\underline{x}) \cap \big(K[\underline{x}] \setminus L(I)\big)$ the set of monomials which are not in the leading ideal of I. Show that B is a K-vector space basis of $K[\underline{x}]/I$.

Exercise 22: [Computation in $K[\underline{x}]/I$ via Normal Forms]

Let $>$ be a *global* monomial ordering on Mon_n^m, $g \in K[\underline{x}]^m$ and $I \leq K[\underline{x}]^m$.

a. If G and G' are two standard bases of I w.r.t. $>$ and $r \in K[\underline{x}]^m$ respectively $r' \in K[\underline{x}]^m$ is the remainder of a *reduced* indeterminate division with remainder of g w.r.t. G respectively G', then $r = r'$.

   We may therefore call $\mathrm{NF}(g, I) := r$ the *normal form* of g w.r.t. I and $>$.

b. $\mathrm{NF}(g, I) + \mathrm{NF}(g', I) = \mathrm{NF}(g + g', I)$ for all $g, g' \in K[\underline{x}]$.

c. $\mathrm{NF}\big(\mathrm{NF}(g, I) \cdot \mathrm{NF}(g', I), I\big) = \mathrm{NF}(g \cdot g', I)$ for all $g, g' \in K[\underline{x}]$.

Exercise 23: Let $f, g \in K[\underline{x}]$. Express $\gcd(f, g)$ and $\mathrm{lcm}(f, g)$ in terms of elements in $\mathrm{syz}(f, g)$ and derive an algorithm to compute these, assuming we can compute a standard basis of $\mathrm{syz}(f, g)$.

Exercise 24: Write a SINGULAR procedure `chaincriterion` which takes a list of pairs of polynomials as input and eliminates pairs using the chain criterion. Then adjust your procedure `standardbasis` with this procedure.