Fachbereich Mathematik — Winter Semester 2007/08, Set 8
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Computer Algebra

Due date: Tuesday, 08/01/2008, 10h00

Exercise 29: Write a SINGULAR procedure `noethernormalisation` which takes as input an ideal I in the polynomial ring $K[\underline{x}]$ and returns a list (M, d) such that $K[x_1, \ldots, x_d] \hookrightarrow K[\underline{x}]/\Phi_{M^{-1}}(I)$ is a Noether normalisation.

Exercise 30: Let R be a ring, $I \trianglelefteq R$, and $f, g \in R$ such that $\langle f, g\rangle = R$ and $f \cdot g \in I$. Then $I = \langle I, f\rangle \cap \langle I, g\rangle$.

Exercise 31: Let $\overline{K}$ the algebraic closure of K, and $I \trianglelefteq K[\underline{x}]$. Show, $I \cdot \overline{K}[\underline{x}] \cap K[\underline{x}] = I$.

Exercise 32: Let K be a field with $\mathrm{char}(K) = 0$ and let $I \trianglelefteq K[\underline{x}]$ with $\dim(K[\underline{x}]/I) = 0$. Show, if $\sqrt{I \cap K[x_i]} = \langle f_i\rangle$, then $\sqrt{I} = I + \langle f_1, \ldots, f_n\rangle$.

Hint, consider a primary decomposition of $(I\langle f_1, \ldots, f_n\rangle) \cdot \overline{K}[\underline{x}]$ induced by factorizing each f_i into linear factors over $\overline{K}$ and applying Exercise 31.