Computer Algebra

Due date: Tuesday, 15/01/2008, 10h00

Exercise 33: Let $I \lhd K[\underline{x}]$ be a zero-dimensional ideal such that $I \cap K[x_n] = \langle f \rangle$ for some irreducible polynomial $f \in K[x_n]$ with $\deg(f) = \dim_K \left(K[\underline{x}]/I\right)$. Show that I is a maximal ideal in general position w.r.t. $>_{lp}$.

Exercise 34: Let K be a field of characteristic zero and let $\mathfrak{m} \lhd \cdot K[\underline{x}]$ be a maximal ideal. Show that there is an irreducible polynomial $f \in K[x_n]$ such that $K[\underline{x}]_{\mathfrak{m}} \cong K[\underline{x}]_{\langle x_1,\ldots,x_{n-1},f\rangle}$.

Exercise 35: Give an example of a zero-dimensional ideal in $\mathbb{Z}/2\mathbb{Z}[x,y]$ which is not in general position w.r.t. the lexicographical ordering with $x > y$.

Exercise 36: Find an algorithm which checks if the ideal in $K[\underline{x}]$ generated by polynomials $f_1,\ldots,f_k$ is zero-dimensional using only a single standard basis computation. Implement this algorithm as `ZeroDimTest` in SINGULAR.