Fachbereich Mathematik
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Winter Semester 2007/08, Set 11
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Computer Algebra

Due date: Tuesday, 29/01/2008, 10h00

Exercise 41: Let $A = (a_{ij}) \in \mathrm{Mat}(m \times n, R)$ be a matrix such that the k-th row is the k-th canonical basis vector in R^n and the l-th column is the l-th canonical basis vector in R^m. And let $A' \in \mathrm{Mat}((m-1) \times (n-1), R)$ be the matrix A with the k-th row and the l-th column removed. Show that A' defines a free presentation of $\mathrm{coker}(A)$.

Exercise 42: Let $(R, \mathfrak{m})$ be a noetherian local ring resp. graded K-algebra and let M be a (graded) finitely generated R-module which has a finite free resolution. Show that every minimal free reolution of M has finite length, and the length of each minimal free resolution of M coincides with the minimum of all lengths of free resolutions of M.

Exercise 43: Compute a minimal free presentation of the R-module

$$M = \left\langle \begin{pmatrix} x^2 \\ y \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix} \right\rangle_R$$

where $R = K[x, y]_{\langle x,y \rangle} / \langle xy \rangle$.

Exercise 44: Use the SINGULAR commands `mres` and `betti` to compute the Betti numbers of $\mathbb{Q} = \mathbb{Q}[x_1, \ldots, x_n]/\langle x_1, \ldots, x_n \rangle$ as $\mathbb{Q}[x_1, \ldots, x_n]$-module for small n. Deduce from the result a conjecture for the general case.