$$\eta_\mu = [u_\mu u_\nu]\, \xi_\nu \quad \text{ist.}$$

Wenn die behauptete Reziprozität besteht, so muss dann folgen:

$$\xi_\nu = (u_\mu u_\nu)\, \eta_\mu$$

Der Beweis ergibt sich durch Rechnung. Wir setzen $\xi_\nu = du_\nu$ berechnen die zugehörigen η_μ und zeigen dann, dass tatsächlich

$$du_\nu = (u_\mu u_\nu)\, \eta_\mu \quad \text{ist.}$$

Denken wir uns zunächst die du_ν der linearen Transformation unterworfen, so wird

$$du_\nu = \frac{\partial u_\nu}{\partial x_\alpha} dx_\alpha + \frac{\partial u_\nu}{\partial y_\alpha} dy_\alpha$$

Demnach wird

$$\begin{aligned}
\eta_\mu &= [u_\mu u_\nu]\left(\frac{\partial u_\nu}{\partial x_\alpha} dx_\alpha + \frac{\partial u_\nu}{\partial y_\alpha} dy_\alpha\right) \\
&= \left(\frac{\partial x_\alpha}{\partial u_\mu}\frac{\partial y_\alpha}{\partial u_\nu} - \frac{\partial x_\alpha}{\partial u_\nu}\frac{\partial y_\alpha}{\partial u_\mu}\right)\left(\frac{\partial u_\nu}{\partial x_\alpha} dx_\alpha + \frac{\partial u_\nu}{\partial y_\alpha} dy_\alpha\right) \\
&= \frac{\partial x_\alpha}{\partial u_\mu}\frac{\partial y_\alpha}{\partial x_\alpha} dx_\alpha + \frac{\partial x_\alpha}{\partial u_\mu}\frac{\partial y_\alpha}{\partial y_\alpha} dy_\alpha - \frac{\partial y_\alpha}{\partial u_\mu}\frac{\partial x_\alpha}{\partial x_\alpha} dx_\alpha - \frac{\partial y_\alpha}{\partial u_\mu}\frac{\partial x_\alpha}{\partial y_\alpha} dy_\alpha \\
&= -\frac{\partial y_\alpha}{\partial u_\mu} dx_\alpha + \frac{\partial x_\alpha}{\partial u_\mu} dy_\alpha
\end{aligned}$$

da

$$\frac{\partial x_\alpha}{\partial x_\alpha} = \frac{\partial y_\alpha}{\partial y_\alpha} = 1 \quad \text{und} \quad \frac{\partial y_\alpha}{\partial x_\alpha} = \frac{\partial x_\alpha}{\partial y_\alpha} = 0 \quad \text{ist.}$$