The Groebner Fan of the Denominator of the j-Invariant of a Curve of Type (4,2)

The computational results listed below prove some of the results in the paper

Eric Katz, Hannah Markwig, Thomas Markwig: The j-invariant ... Preprint 2007.

For an explantion of the terms used in the following, for some of the notation, like $c_T(i,j)$ or $\eta_T(i,j)$, and for a thorough explanation of the results we refer to this paper.

If we consider a curve given by the general polynomial

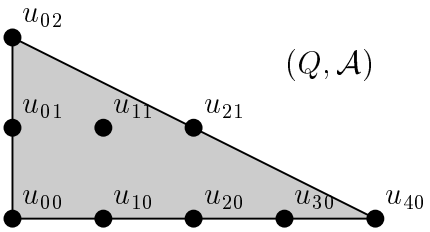
$$f = u_{40}x^4 + u_{30}x^3 + u_{21}x^2y + u_{20}x^2 + u_{11}xy + u_{02}y^2 + u_{10}x + u_{01}y + u_{00};$$

then the j-invariant of f is a quotient of two homogeneous polynomials of degree 12, where the numerator has 267 terms and the denominator has 312 terms in the unknowns

$$u_{11}, u_{40}, u_{30}, u_{20}, u_{10}, u_{00}, u_{21}, u_{01}, u_{02}.$$

Here we just want to consider the denominator Δ, which is u_{02}^2 times the discriminant of f, and its Newton polytope $N(\Delta)$. With the help of `polymake` we computed the 46 vertices of $N(\Delta)$ and the corresponding edges. Fixing one vertex and subtracting from it the vertices to which it is connected gives inequalities for the dual cone corresponding to this vertex in the Groebner fan of Δ, which is the normal fan dual to $N(\Delta)$. These are precisely the inequalities which the valuations of the coefficients of f have to fulfill if the corresponding vertex of the Newton polytope $N(\Delta)$ is the leading term of Δ of f in the sense that its valuation is minimal. (We do not distinguish between the coefficients and their valuaions when writing down inequalities!)

In what follows we give for each vertex in $N(\Delta)$ the coarsest Newton subdivision whose cone in the secondary fan of



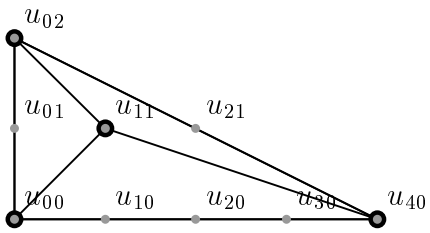
is contained in the cone of $N(\Delta)$ dual to this vertex. This should be interpreted as follows: the vertices which correspond to a fat black dot are those whose coefficient occurs in the corresponding vertex of $N(\Delta)$; the black inner edges are those which are expected to be present, since due to the given inequalities all crossing edges lie beneath these.

Moreover, we check that the cycle length of a tropical curve with this Newton subdivison is the negative of the generic valuation of the j-invariant in that case if the tropical curve has a cycle. Note that only the first 24 vertices of $N(\Delta)$ correspond to tropical curves which have a cycle!

Finally, we give for each vertex in the Newton polytope $N(\Delta)$ a list of inequalities which describe the corresponding dual cone in the Groebner fan of Δ. Here one should in principle replace the coefficients by their valuation.

1. Vertex: $u_{11}^8 u_{40} u_{00} u_{02}^2 \equiv$ (8,1,0,0,0,1,0,0,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $4 \cdot u_{11} - u_{40} - u_{00} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $4 \cdot u_{11} - u_{40} - u_{00} - 2 \cdot u_{02}$

2	2				
1	0	-4	0		
0	1	0	0	0	1
$c_T(i,j)$	0	1	2	3	4

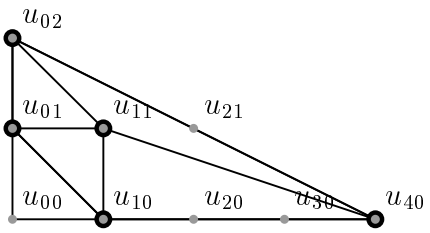
2	1				
1	0	8	0		
0	1	0	0	0	1
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$
\begin{aligned}
u_{11} + u_{00} &< u_{10} + u_{01} \\
u_{11} + u_{40} &< u_{30} + u_{21} \\
2 \cdot u_{11} + u_{00} &< u_{20} + 2 \cdot u_{01} \\
2 \cdot u_{11} + u_{00} &< 2 \cdot u_{10} + u_{02} \\
2 \cdot u_{11} + u_{40} &< 2 \cdot u_{30} + u_{02} \\
2 \cdot u_{11} + u_{40} &< u_{20} + 2 \cdot u_{21} \\
3 \cdot u_{11} + u_{00} &< u_{30} + 3 \cdot u_{01} \\
3 \cdot u_{11} + u_{40} &< u_{10} + 3 \cdot u_{21} \\
4 \cdot u_{11} + u_{00} &< u_{40} + 4 \cdot u_{01} \\
4 \cdot u_{11} + u_{40} &< u_{00} + 4 \cdot u_{21} \\
8 \cdot u_{11} &< 2 \cdot u_{40} + 2 \cdot u_{00} + 4 \cdot u_{02} \\
8 \cdot u_{11} &< 4 \cdot u_{20} + 4 \cdot u_{02}
\end{aligned}
$$

2. Vertex: $u_{11}^7 u_{40} u_{10} u_{01} u_{02}^2 \equiv$ (7,1,0,0,1,0,0,1,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $5 \cdot u_{11} - u_{40} - u_{10} - u_{01} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $5 \cdot u_{11} - u_{40} - u_{10} - u_{01} - 2 \cdot u_{02}$

2	2				
1	1	-5	0		
0	0	1	0	0	1
$c_T(i,j)$	0	1	2	3	4

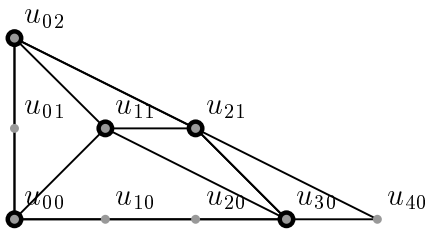
2	1				
1	1	7	0		
0	0	1	0	0	1
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{10} + u_{01} &< u_{11} + u_{00} \\
u_{11} + u_{10} &< u_{20} + u_{01} \\
u_{11} + u_{01} &< u_{10} + u_{02} \\
u_{11} + u_{40} &< u_{30} + u_{21} \\
2 \cdot u_{11} + u_{10} &< u_{30} + 2 \cdot u_{01} \\
2 \cdot u_{11} + u_{40} &< 2 \cdot u_{30} + u_{02} \\
2 \cdot u_{11} + u_{40} &< u_{20} + 2 \cdot u_{21} \\
3 \cdot u_{11} + u_{10} &< u_{40} + 3 \cdot u_{01} \\
3 \cdot u_{11} + u_{40} &< u_{10} + 3 \cdot u_{21}
\end{aligned}$$

3. Vertex: $u_{11}^7 u_{30} u_{00} u_{21} u_{02}^2 \equiv$ (7,0,1,0,0,1,1,0,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $5 \cdot u_{11} - u_{30} - u_{00} - u_{21} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $5 \cdot u_{11} - u_{30} - u_{00} - u_{21} - 2 \cdot u_{02}$

2	2				
1	0	-5	1		
0	1	0	0	1	0
$c_T(i,j)$	0	1	2	3	4

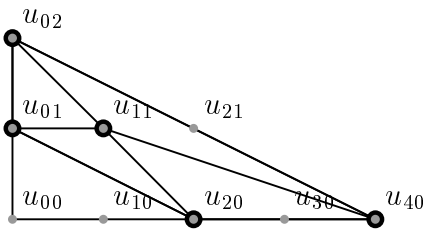
2	1				
1	0	7	1		
0	1	0	0	1	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{30} + u_{21} &< u_{11} + u_{40} \\
u_{11} + u_{21} &< u_{30} + u_{02} \\
u_{11} + u_{00} &< u_{10} + u_{01} \\
u_{11} + u_{30} &< u_{20} + u_{21} \\
2 \cdot u_{11} + u_{00} &< u_{20} + 2 \cdot u_{01} \\
2 \cdot u_{11} + u_{00} &< 2 \cdot u_{10} + u_{02} \\
2 \cdot u_{11} + u_{30} &< u_{10} + 2 \cdot u_{21} \\
3 \cdot u_{11} + u_{00} &< u_{30} + 3 \cdot u_{01} \\
3 \cdot u_{11} + u_{30} &< u_{00} + 3 \cdot u_{21}
\end{aligned}$$

4. Vertex: $u_{11}^6 u_{40} u_{20} u_{01}^2 u_{02}^2 \equiv$ (6,1,0,1,0,0,0,2,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $6 \cdot u_{11} - u_{40} - u_{20} - 2 \cdot u_{01} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $6 \cdot u_{11} - u_{40} - u_{20} - 2 \cdot u_{01} - 2 \cdot u_{02}$

2	2				
1	2	-6	0		
0	0	0	1	0	1
$c_T(i,j)$	0	1	2	3	4

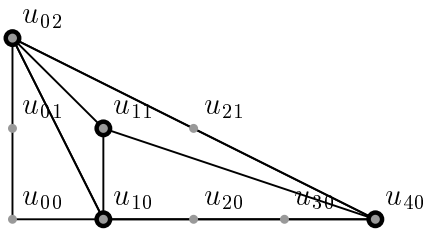
2	1				
1	2	6	0		
0	0	0	1	0	1
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{20} + 2 \cdot u_{01} &< 2 \cdot u_{11} + u_{00} \\
u_{20} + u_{01} &< u_{11} + u_{10} \\
u_{11} + u_{20} &< u_{30} + u_{01} \\
u_{11} + u_{40} &< u_{30} + u_{21} \\
2 \cdot u_{11} + u_{20} &< u_{40} + 2 \cdot u_{01} \\
2 \cdot u_{11} + u_{40} &< 2 \cdot u_{30} + u_{02} \\
2 \cdot u_{11} + u_{40} &< u_{20} + 2 \cdot u_{21} \\
6 \cdot u_{11} &< 3 \cdot u_{20} + 3 \cdot u_{02}
\end{aligned}$$

5. Vertex: $u_{11}^6 u_{40} u_{10}^2 u_{02}^3 \equiv$ (6,1,0,0,2,0,0,0,3)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $6 \cdot u_{11} - u_{40} - 2 \cdot u_{10} - 3 \cdot u_{02}$

The generic valuation of the j-invariant is: $6 \cdot u_{11} - u_{40} - 2 \cdot u_{10} - 3 \cdot u_{02}$

2	3				
1	0	-6	0		
0	0	2	0	0	1
$c_T(i,j)$	0	1	2	3	4

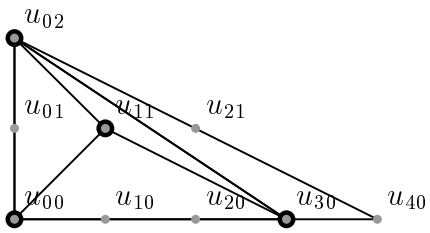
2	2				
1	0	6	0		
0	0	2	0	0	1
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
2 \cdot u_{10} + u_{02} &< 2 \cdot u_{11} + u_{00} \\
u_{10} + u_{02} &< u_{11} + u_{01} \\
u_{11} + u_{40} &< u_{30} + u_{21} \\
2 \cdot u_{11} + u_{40} &< 2 \cdot u_{30} + u_{02} \\
2 \cdot u_{11} + u_{40} &< u_{20} + 2 \cdot u_{21} \\
3 \cdot u_{11} + u_{40} &< u_{10} + 3 \cdot u_{21} \\
6 \cdot u_{11} &< u_{40} + 2 \cdot u_{10} + 3 \cdot u_{02} \\
6 \cdot u_{11} &< 3 \cdot u_{20} + 3 \cdot u_{02}
\end{aligned}$$

6. Vertex: $u_{11}^6 u_{30}^2 u_{00} u_{02}^3 \equiv$ (6,0,2,0,0,1,0,0,3)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $6 \cdot u_{11} - 2 \cdot u_{30} - u_{00} - 3 \cdot u_{02}$

The generic valuation of the j-invariant is: $6 \cdot u_{11} - 2 \cdot u_{30} - u_{00} - 3 \cdot u_{02}$

2	3				
1	0	-6	0		
0	1	0	0	2	0
$c_T(i,j)$	0	1	2	3	4

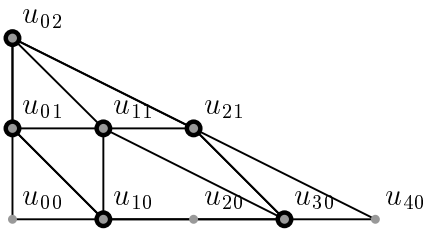
2	2				
1	0	6	0		
0	1	0	0	2	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
2 \cdot u_{30} + u_{02} &< 2 \cdot u_{11} + u_{40} \\
u_{30} + u_{02} &< u_{11} + u_{21} \\
u_{11} + u_{00} &< u_{10} + u_{01} \\
2 \cdot u_{11} + u_{00} &< u_{20} + 2 \cdot u_{01} \\
2 \cdot u_{11} + u_{00} &< 2 \cdot u_{10} + u_{02} \\
3 \cdot u_{11} + u_{00} &< u_{30} + 3 \cdot u_{01} \\
6 \cdot u_{11} &< 2 \cdot u_{30} + u_{00} + 3 \cdot u_{02} \\
6 \cdot u_{11} &< 3 \cdot u_{20} + 3 \cdot u_{02}
\end{aligned}$$

7. Vertex: $u_{11}^6 u_{30} u_{10} u_{21} u_{01} u_{02}^2 \equiv$ (6,0,1,0,1,0,1,1,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $6 \cdot u_{11} - u_{30} - u_{10} - u_{21} - u_{01} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $6 \cdot u_{11} - u_{30} - u_{10} - u_{21} - u_{01} - 2 \cdot u_{02}$

2	2				
1	1	-6	1		
0	0	1	0	1	0
$c_T(i,j)$	0	1	2	3	4

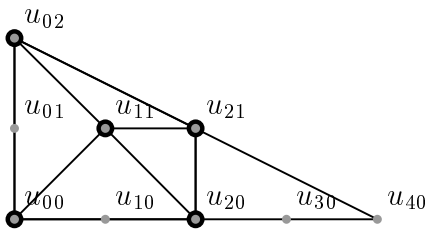
2	1				
1	1	6	1		
0	0	1	0	1	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{30} + u_{21} &< u_{11} + u_{40} \\
u_{10} + u_{01} &< u_{11} + u_{00} \\
u_{11} + u_{21} &< u_{30} + u_{02} \\
u_{11} + u_{10} &< u_{20} + u_{01} \\
u_{11} + u_{01} &< u_{10} + u_{02} \\
u_{11} + u_{30} &< u_{20} + u_{21} \\
2 \cdot u_{11} + u_{10} &< u_{30} + 2 \cdot u_{01} \\
2 \cdot u_{11} + u_{30} &< u_{10} + 2 \cdot u_{21}
\end{aligned}$$

8. Vertex: $u_{11}^6 u_{20} u_{00} u_{21}^2 u_{02}^2 \equiv$ (6,0,0,1,0,1,2,0,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $6 \cdot u_{11} - u_{20} - u_{00} - 2 \cdot u_{21} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $6 \cdot u_{11} - u_{20} - u_{00} - 2 \cdot u_{21} - 2 \cdot u_{02}$

2	2				
1	0	-6	2		
0	1	0	1	0	0
$c_T(i,j)$	0	1	2	3	4

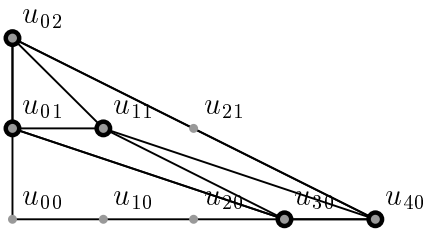
2	1				
1	0	6	2		
0	1	0	1	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{20} + 2 \cdot u_{21} &< 2 \cdot u_{11} + u_{40} \\
u_{20} + u_{21} &< u_{11} + u_{30} \\
u_{11} + u_{00} &< u_{10} + u_{01} \\
u_{11} + u_{20} &< u_{10} + u_{21} \\
2 \cdot u_{11} + u_{00} &< u_{20} + 2 \cdot u_{01} \\
2 \cdot u_{11} + u_{00} &< 2 \cdot u_{10} + u_{02} \\
2 \cdot u_{11} + u_{20} &< u_{00} + 2 \cdot u_{21} \\
6 \cdot u_{11} &< 3 \cdot u_{20} + 3 \cdot u_{02}
\end{aligned}$$

9. Vertex: $u_{11}^5 u_{40} u_{30} u_{01}^3 u_{02}^2 \equiv$ (5,1,1,0,0,0,0,3,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $7 \cdot u_{11} - u_{40} - u_{30} - 3 \cdot u_{01} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $7 \cdot u_{11} - u_{40} - u_{30} - 3 \cdot u_{01} - 2 \cdot u_{02}$

2	2				
1	3	-7	0		
0	0	0	0	1	1
$c_T(i,j)$	0	1	2	3	4

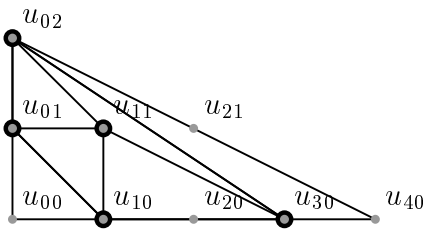
2	1				
1	3	5	0		
0	0	0	0	1	1
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{30} + 3 \cdot u_{01} &< 3 \cdot u_{11} + u_{00} \\
u_{30} + 2 \cdot u_{01} &< 2 \cdot u_{11} + u_{10} \\
u_{30} + u_{01} &< u_{11} + u_{20} \\
u_{11} + u_{30} &< u_{40} + u_{01} \\
u_{11} + u_{40} &< u_{30} + u_{21} \\
2 \cdot u_{11} + u_{40} &< 2 \cdot u_{30} + u_{02}
\end{aligned}$$

10. Vertex: $u_{11}^5 u_{30}^2 u_{10} u_{01} u_{02}^3 \equiv$ (5,0,2,0,1,0,0,1,3)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $7 \cdot u_{11} - 2 \cdot u_{30} - u_{10} - u_{01} - 3 \cdot u_{02}$

The generic valuation of the j-invariant is: $7 \cdot u_{11} - 2 \cdot u_{30} - u_{10} - u_{01} - 3 \cdot u_{02}$

2	3				
1	1	-7	0		
0	0	1	0	2	0
$c_T(i,j)$	0	1	2	3	4

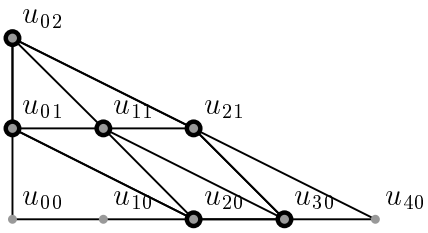
2	2				
1	1	5	0		
0	0	1	0	2	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
2 \cdot u_{30} + u_{02} &< 2 \cdot u_{11} + u_{40} \\
u_{10} + u_{01} &< u_{11} + u_{00} \\
u_{30} + u_{02} &< u_{11} + u_{21} \\
u_{11} + u_{10} &< u_{20} + u_{01} \\
u_{11} + u_{01} &< u_{10} + u_{02} \\
2 \cdot u_{11} + u_{10} &< u_{30} + 2 \cdot u_{01}
\end{aligned}$$

11. Vertex: $u_{11}^5 u_{30} u_{20} u_{21} u_{01}^2 u_{02}^2 \equiv$ (5,0,1,1,0,0,1,2,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $7 \cdot u_{11} - u_{30} - u_{20} - u_{21} - 2 \cdot u_{01} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $7 \cdot u_{11} - u_{30} - u_{20} - u_{21} - 2 \cdot u_{01} - 2 \cdot u_{02}$

2	2				
1	2	-7	1		
0	0	0	1	1	0
$c_T(i,j)$	0	1	2	3	4

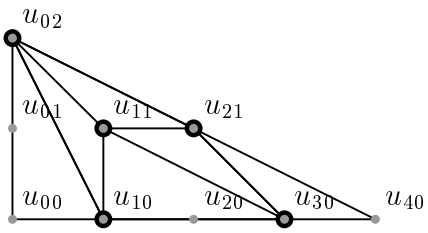
2	1				
1	2	5	1		
0	0	0	1	1	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned} u_{20} + 2 \cdot u_{01} &< 2 \cdot u_{11} + u_{00} \\ u_{30} + u_{21} &< u_{11} + u_{40} \\ u_{20} + u_{01} &< u_{11} + u_{10} \\ u_{11} + u_{21} &< u_{30} + u_{02} \\ u_{11} + u_{20} &< u_{30} + u_{01} \\ u_{11} + u_{30} &< u_{20} + u_{21} \end{aligned}$$

12. Vertex: $u_{11}^5 u_{30} u_{10}^2 u_{21} u_{02}^3 \equiv$ (5,0,1,0,2,0,1,0,3)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $7 \cdot u_{11} - u_{30} - 2 \cdot u_{10} - u_{21} - 3 \cdot u_{02}$

The generic valuation of the j-invariant is: $7 \cdot u_{11} - u_{30} - 2 \cdot u_{10} - u_{21} - 3 \cdot u_{02}$

2	3				
1	0	-7	1		
0	0	2	0	1	0
$c_T(i,j)$	0	1	2	3	4

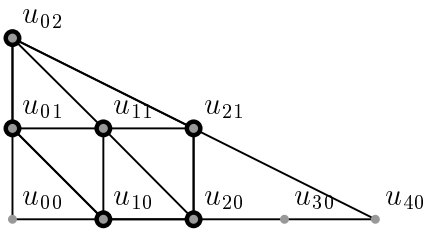
2	2				
1	0	5	1		
0	0	2	0	1	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
2 \cdot u_{10} + u_{02} &< 2 \cdot u_{11} + u_{00} \\
u_{30} + u_{21} &< u_{11} + u_{40} \\
u_{10} + u_{02} &< u_{11} + u_{01} \\
u_{11} + u_{21} &< u_{30} + u_{02} \\
u_{11} + u_{30} &< u_{20} + u_{21} \\
2 \cdot u_{11} + u_{30} &< u_{10} + 2 \cdot u_{21}
\end{aligned}$$

13. Vertex: $u_{11}^5 u_{20} u_{10} u_{21}^2 u_{01} u_{02}^2 \equiv$ (5,0,0,1,1,0,2,1,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $7 \cdot u_{11} - u_{20} - u_{10} - 2 \cdot u_{21} - u_{01} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $7 \cdot u_{11} - u_{20} - u_{10} - 2 \cdot u_{21} - u_{01} - 2 \cdot u_{02}$

2	2				
1	1	-7	2		
0	0	1	1	0	0
$c_T(i,j)$	0	1	2	3	4

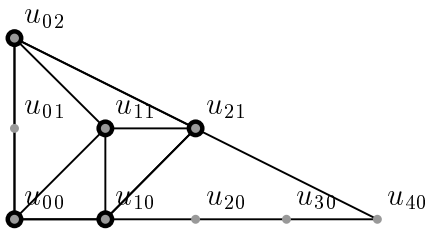
2	1				
1	1	5	2		
0	0	1	1	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned} u_{20} + 2 \cdot u_{21} &< 2 \cdot u_{11} + u_{40} \\ u_{20} + u_{21} &< u_{11} + u_{30} \\ u_{10} + u_{01} &< u_{11} + u_{00} \\ u_{11} + u_{10} &< u_{20} + u_{01} \\ u_{11} + u_{01} &< u_{10} + u_{02} \\ u_{11} + u_{20} &< u_{10} + u_{21} \end{aligned}$$

14. Vertex: $u_{11}^5 u_{10} u_{00} u_{21}^3 u_{02}^2 \equiv$ (5,0,0,0,1,1,3,0,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $7 \cdot u_{11} - u_{10} - u_{00} - 3 \cdot u_{21} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $7 \cdot u_{11} - u_{10} - u_{00} - 3 \cdot u_{21} - 2 \cdot u_{02}$

2	2				
1	0	-7	3		
0	1	1	0	0	0
$c_T(i,j)$	0	1	2	3	4

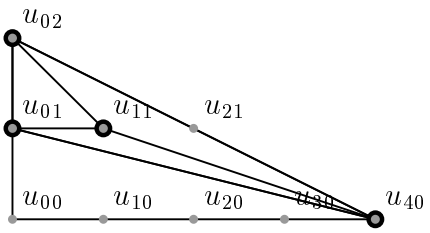
2	1				
1	0	5	3		
0	1	1	0	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{10} + 3 \cdot u_{21} &< 3 \cdot u_{11} + u_{40} \\
u_{10} + 2 \cdot u_{21} &< 2 \cdot u_{11} + u_{30} \\
u_{10} + u_{21} &< u_{11} + u_{20} \\
u_{11} + u_{00} &< u_{10} + u_{01} \\
u_{11} + u_{10} &< u_{00} + u_{21} \\
2 \cdot u_{11} + u_{00} &< 2 \cdot u_{10} + u_{02}
\end{aligned}$$

15. Vertex: $u_{11}^4 u_{40}^2 u_{01}^4 u_{02}^2 \equiv$ (4,2,0,0,0,0,0,4,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $8 \cdot u_{11} - 2 \cdot u_{40} - 4 \cdot u_{01} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $8 \cdot u_{11} - 2 \cdot u_{40} - 4 \cdot u_{01} - 2 \cdot u_{02}$

2	2				
1	4	-8	0		
0	0	0	0	0	2
$c_T(i,j)$	0	1	2	3	4

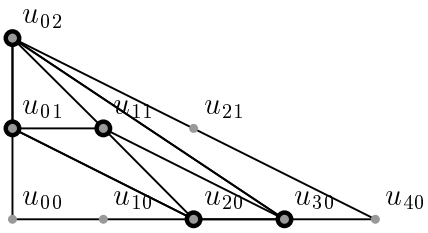
2	1				
1	4	4	0		
0	0	0	0	0	2
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{40} + 4 \cdot u_{01} &< 4 \cdot u_{11} + u_{00} \\
u_{40} + 3 \cdot u_{01} &< 3 \cdot u_{11} + u_{10} \\
u_{40} + 2 \cdot u_{01} &< 2 \cdot u_{11} + u_{20} \\
u_{40} + u_{01} &< u_{11} + u_{30} \\
4 \cdot u_{11} &< u_{40} + 2 \cdot u_{01} + u_{02} \\
4 \cdot u_{11} &< 2 \cdot u_{21} + 2 \cdot u_{01}
\end{aligned}$$

16. Vertex: $u_{11}^4 u_{30}^2 u_{20} u_{01}^2 u_{02}^3 \equiv$ (4,0,2,1,0,0,0,2,3)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $8 \cdot u_{11} - 2 \cdot u_{30} - u_{20} - 2 \cdot u_{01} - 3 \cdot u_{02}$

The generic valuation of the j-invariant is: $8 \cdot u_{11} - 2 \cdot u_{30} - u_{20} - 2 \cdot u_{01} - 3 \cdot u_{02}$

2	3				
1	2	-8	0		
0	0	0	1	2	0
$c_T(i,j)$	0	1	2	3	4

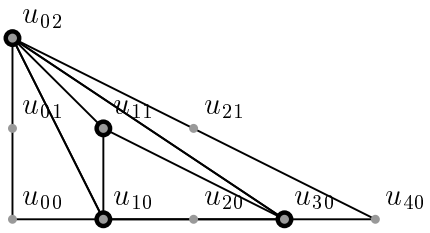
2	2				
1	2	4	0		
0	0	0	1	2	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned} 2 \cdot u_{30} + u_{02} &< 2 \cdot u_{11} + u_{40} \\ u_{20} + 2 \cdot u_{01} &< 2 \cdot u_{11} + u_{00} \\ u_{20} + u_{01} &< u_{11} + u_{10} \\ u_{30} + u_{02} &< u_{11} + u_{21} \\ u_{11} + u_{20} &< u_{30} + u_{01} \\ 4 \cdot u_{11} &< 2 \cdot u_{20} + 2 \cdot u_{02} \end{aligned}$$

17. Vertex: $u_{11}^4 u_{30}^2 u_{10}^2 u_{02}^4 \equiv$ (4,0,2,0,2,0,0,0,4)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $8 \cdot u_{11} - 2 \cdot u_{30} - 2 \cdot u_{10} - 4 \cdot u_{02}$

The generic valuation of the j-invariant is: $8 \cdot u_{11} - 2 \cdot u_{30} - 2 \cdot u_{10} - 4 \cdot u_{02}$

2	4				
1	0	-8	0		
0	0	2	0	2	0
$c_T(i,j)$	0	1	2	3	4

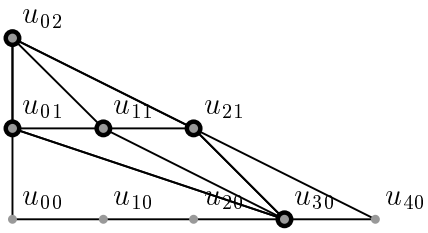
2	3				
1	0	4	0		
0	0	2	0	2	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
2 \cdot u_{30} + u_{02} &< 2 \cdot u_{11} + u_{40} \\
2 \cdot u_{10} + u_{02} &< 2 \cdot u_{11} + u_{00} \\
u_{10} + u_{02} &< u_{11} + u_{01} \\
u_{30} + u_{02} &< u_{11} + u_{21} \\
4 \cdot u_{11} &< u_{30} + u_{10} + 2 \cdot u_{02} \\
4 \cdot u_{11} &< 2 \cdot u_{20} + 2 \cdot u_{02}
\end{aligned}$$

18. Vertex: $u_{11}^4 u_{30}^2 u_{21} u_{01}^3 u_{02}^2 \equiv$ (4,0,2,0,0,0,1,3,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $8 \cdot u_{11} - 2 \cdot u_{30} - u_{21} - 3 \cdot u_{01} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $8 \cdot u_{11} - 2 \cdot u_{30} - u_{21} - 3 \cdot u_{01} - 2 \cdot u_{02}$

2	2				
1	3	-8	1		
0	0	0	0	2	0
$c_T(i,j)$	0	1	2	3	4

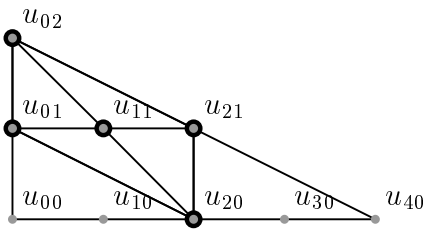
2	1				
1	3	4	1		
0	0	0	0	2	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned} u_{30} + 3 \cdot u_{01} &< 3 \cdot u_{11} + u_{00} \\ u_{30} + 2 \cdot u_{01} &< 2 \cdot u_{11} + u_{10} \\ u_{30} + u_{21} &< u_{11} + u_{40} \\ u_{30} + u_{01} &< u_{11} + u_{20} \\ u_{11} + u_{21} &< u_{30} + u_{02} \\ 4 \cdot u_{11} &< 2 \cdot u_{21} + 2 \cdot u_{01} \end{aligned}$$

19. Vertex: $u_{11}^4 u_{20}^2 u_{21}^2 u_{01}^2 u_{02}^2 \equiv$ (4,0,0,2,0,0,2,2,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $8 \cdot u_{11} - 2 \cdot u_{20} - 2 \cdot u_{21} - 2 \cdot u_{01} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $8 \cdot u_{11} - 2 \cdot u_{20} - 2 \cdot u_{21} - 2 \cdot u_{01} - 2 \cdot u_{02}$

2	2				
1	2	-8	2		
0	0	0	2	0	0
$c_T(i,j)$	0	1	2	3	4

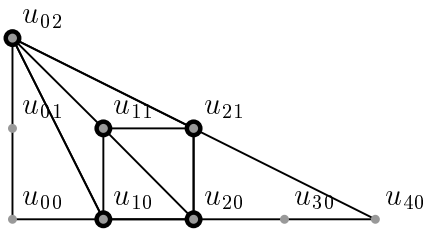
2	1				
1	2	4	2		
0	0	0	2	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{20} + 2 \cdot u_{21} &< 2 \cdot u_{11} + u_{40} \\
u_{20} + 2 \cdot u_{01} &< 2 \cdot u_{11} + u_{00} \\
u_{20} + u_{21} &< u_{11} + u_{30} \\
u_{20} + u_{01} &< u_{11} + u_{10} \\
4 \cdot u_{11} &< 2 \cdot u_{20} + 2 \cdot u_{02} \\
4 \cdot u_{11} &< 2 \cdot u_{21} + 2 \cdot u_{01}
\end{aligned}$$

20. Vertex: $u_{11}^4 u_{20} u_{10}^2 u_{21}^2 u_{02}^3 \equiv$ (4,0,0,1,2,0,2,0,3)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $8 \cdot u_{11} - u_{20} - 2 \cdot u_{10} - 2 \cdot u_{21} - 3 \cdot u_{02}$

The generic valuation of the j-invariant is: $8 \cdot u_{11} - u_{20} - 2 \cdot u_{10} - 2 \cdot u_{21} - 3 \cdot u_{02}$

2	3				
1	0	-8	2		
0	0	2	1	0	0
$c_T(i,j)$	0	1	2	3	4

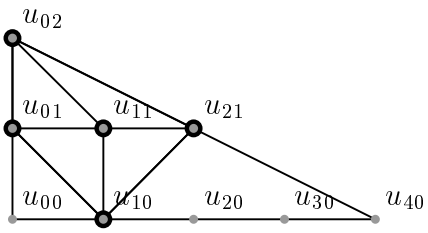
2	2				
1	0	4	2		
0	0	2	1	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned} u_{20} + 2 \cdot u_{21} &< 2 \cdot u_{11} + u_{40} \\ 2 \cdot u_{10} + u_{02} &< 2 \cdot u_{11} + u_{00} \\ u_{20} + u_{21} &< u_{11} + u_{30} \\ u_{10} + u_{02} &< u_{11} + u_{01} \\ u_{11} + u_{20} &< u_{10} + u_{21} \\ 4 \cdot u_{11} &< 2 \cdot u_{20} + 2 \cdot u_{02} \end{aligned}$$

21. Vertex: $u_{11}^4 u_{10}^2 u_{21}^3 u_{01} u_{02}^2 \equiv$ (4,0,0,0,2,0,3,1,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $8 \cdot u_{11} - 2 \cdot u_{10} - 3 \cdot u_{21} - u_{01} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $8 \cdot u_{11} - 2 \cdot u_{10} - 3 \cdot u_{21} - u_{01} - 2 \cdot u_{02}$

2	2				
1	1	-8	3		
0	0	2	0	0	0
$c_T(i,j)$	0	1	2	3	4

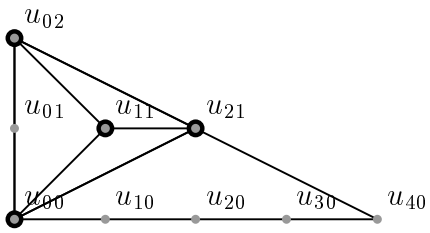
2	1				
1	1	4	3		
0	0	2	0	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned} u_{10} + 3 \cdot u_{21} &< 3 \cdot u_{11} + u_{40} \\ u_{10} + 2 \cdot u_{21} &< 2 \cdot u_{11} + u_{30} \\ u_{10} + u_{21} &< u_{11} + u_{20} \\ u_{10} + u_{01} &< u_{11} + u_{00} \\ u_{11} + u_{01} &< u_{10} + u_{02} \\ 4 \cdot u_{11} &< 2 \cdot u_{21} + 2 \cdot u_{01} \end{aligned}$$

22. Vertex: $u_{11}^4 u_{00}^2 u_{21}^4 u_{02}^2 \equiv$ (4,0,0,0,0,2,4,0,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $8 \cdot u_{11} - 2 \cdot u_{00} - 4 \cdot u_{21} - 2 \cdot u_{02}$

The generic valuation of the j-invariant is: $8 \cdot u_{11} - 2 \cdot u_{00} - 4 \cdot u_{21} - 2 \cdot u_{02}$

2	2				
1	0	-8	4		
0	2	0	0	0	0
$c_T(i,j)$	0	1	2	3	4

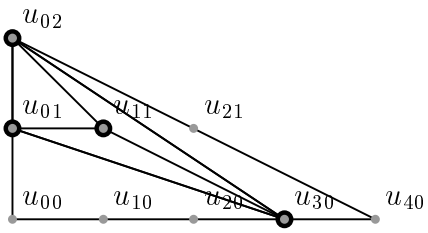
2	1				
1	0	4	4		
0	2	0	0	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{00} + 4 \cdot u_{21} &< 4 \cdot u_{11} + u_{40} \\
u_{00} + 3 \cdot u_{21} &< 3 \cdot u_{11} + u_{30} \\
u_{00} + 2 \cdot u_{21} &< 2 \cdot u_{11} + u_{20} \\
u_{00} + u_{21} &< u_{11} + u_{10} \\
4 \cdot u_{11} &< u_{00} + 2 \cdot u_{21} + u_{02} \\
4 \cdot u_{11} &< 2 \cdot u_{21} + 2 \cdot u_{01}
\end{aligned}$$

23. Vertex: $u_{11}^3 u_{30}^3 u_{01}^3 u_{02}^3 \equiv$ (3,0,3,0,0,0,0,3,3)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $9 \cdot u_{11} - 3 \cdot u_{30} - 3 \cdot u_{01} - 3 \cdot u_{02}$

The generic valuation of the j-invariant is: $9 \cdot u_{11} - 3 \cdot u_{30} - 3 \cdot u_{01} - 3 \cdot u_{02}$

2	3				
1	3	-9	0		
0	0	0	0	3	0
$c_T(i,j)$	0	1	2	3	4

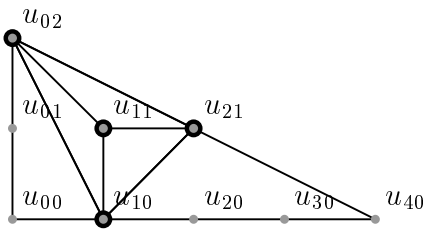
2	2				
1	3	3	0		
0	0	0	0	3	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{30} + 3 \cdot u_{01} &< 3 \cdot u_{11} + u_{00} \\
2 \cdot u_{30} + u_{02} &< 2 \cdot u_{11} + u_{40} \\
u_{30} + 2 \cdot u_{01} &< 2 \cdot u_{11} + u_{10} \\
u_{30} + u_{01} &< u_{11} + u_{20} \\
u_{30} + u_{02} &< u_{11} + u_{21} \\
3 \cdot u_{11} &< u_{30} + u_{01} + u_{02}
\end{aligned}$$

24. Vertex: $u_{11}^3 u_{10}^3 u_{21}^3 u_{02}^3 \equiv$ (3,0,0,0,3,0,3,0,3)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: $9 \cdot u_{11} - 3 \cdot u_{10} - 3 \cdot u_{21} - 3 \cdot u_{02}$

The generic valuation of the j-invariant is: $9 \cdot u_{11} - 3 \cdot u_{10} - 3 \cdot u_{21} - 3 \cdot u_{02}$

2	3				
1	0	-9	3		
0	0	3	0	0	0
$c_T(i,j)$	0	1	2	3	4

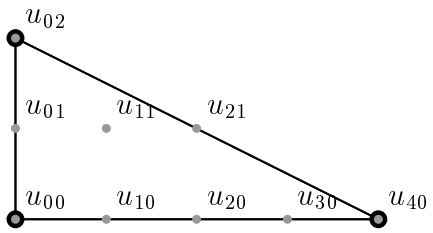
2	2				
1	0	3	3		
0	0	3	0	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{10} + 3 \cdot u_{21} &< 3 \cdot u_{11} + u_{40} \\
u_{10} + 2 \cdot u_{21} &< 2 \cdot u_{11} + u_{30} \\
2 \cdot u_{10} + u_{02} &< 2 \cdot u_{11} + u_{00} \\
u_{10} + u_{21} &< u_{11} + u_{20} \\
u_{10} + u_{02} &< u_{11} + u_{01} \\
3 \cdot u_{11} &< u_{10} + u_{21} + u_{02}
\end{aligned}$$

25. Vertex: $u_{40}^3 u_{00}^3 u_{02}^6 \equiv$ (0,3,0,0,0,3,0,0,6)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $3 \cdot u_{40} + 3 \cdot u_{00} + 6 \cdot u_{02}$

2	2				
1	0	-4	0		
0	1	0	0	0	1
$c_T(i,j)$	0	1	2	3	4

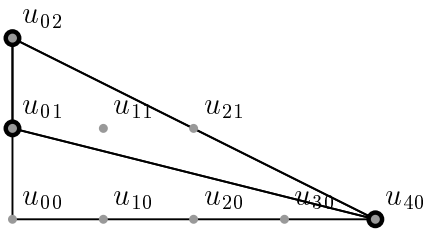
2	5				
1	0	0	0		
0	3	0	0	0	3
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned} 2 \cdot u_{40} + 2 \cdot u_{00} + 4 \cdot u_{02} &< 8 \cdot u_{11} \\ 3 \cdot u_{00} + 3 \cdot u_{02} &< 6 \cdot u_{01} \\ u_{40} + 3 \cdot u_{00} &< 4 \cdot u_{10} \\ 2 \cdot u_{40} + 2 \cdot u_{00} &< 4 \cdot u_{20} \\ 3 \cdot u_{40} + u_{00} &< 4 \cdot u_{30} \\ 3 \cdot u_{40} + 3 \cdot u_{02} &< 6 \cdot u_{21} \end{aligned}$$

26. Vertex: $u_{40}^3 u_{01}^6 u_{02}^3 \equiv$ (0,3,0,0,0,0,0,6,3)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $3 \cdot u_{40} + 6 \cdot u_{01} + 3 \cdot u_{02}$

2	2				
1	4	-8	0		
0	0	0	0	0	2
$c_T(i,j)$	0	1	2	3	4

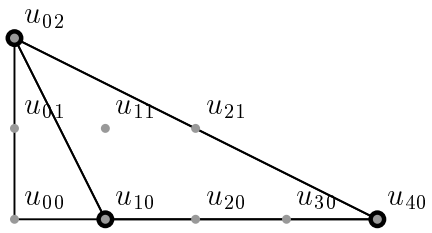
2	2				
1	6	0	0		
0	0	0	0	0	3
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{40} + 2 \cdot u_{01} + u_{02} &< 4 \cdot u_{11} \\
6 \cdot u_{01} &< 3 \cdot u_{00} + 3 \cdot u_{02} \\
u_{40} + 6 \cdot u_{01} &< 4 \cdot u_{10} + 3 \cdot u_{02} \\
u_{40} + u_{02} &< 2 \cdot u_{21} \\
2 \cdot u_{40} + 4 \cdot u_{01} &< 4 \cdot u_{20} + 2 \cdot u_{02} \\
3 \cdot u_{40} + 2 \cdot u_{01} &< 4 \cdot u_{30} + u_{02}
\end{aligned}$$

27. Vertex: $u_{40}^2 u_{10}^4 u_{02}^6 \equiv$ (0,2,0,0,4,0,0,0,6)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $2 \cdot u_{40} + 4 \cdot u_{10} + 6 \cdot u_{02}$

2	3				
1	0	-6	0		
0	0	2	0	0	1
$c_T(i,j)$	0	1	2	3	4

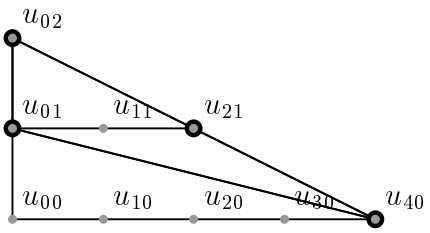
2	5				
1	0	0	0		
0	0	4	0	0	2
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned} u_{40} + 2 \cdot u_{10} + 3 \cdot u_{02} &< 6 \cdot u_{11} \\ 4 \cdot u_{10} &< u_{40} + 3 \cdot u_{00} \\ 4 \cdot u_{10} + 3 \cdot u_{02} &< u_{40} + 6 \cdot u_{01} \\ u_{40} + 2 \cdot u_{10} &< 3 \cdot u_{20} \\ 2 \cdot u_{40} + u_{10} &< 3 \cdot u_{30} \\ 2 \cdot u_{40} + 2 \cdot u_{02} &< 4 \cdot u_{21} \end{aligned}$$

28. Vertex: $u_{40}^2 u_{21}^2 u_{01}^6 u_{02}^2 \equiv$ (0,2,0,0,0,0,2,6,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $2 \cdot u_{40} + 2 \cdot u_{21} + 6 \cdot u_{01} + 2 \cdot u_{02}$

2	2				
1	4	-8	0		
0	0	0	0	0	2
$c_T(i,j)$	0	1	2	3	4

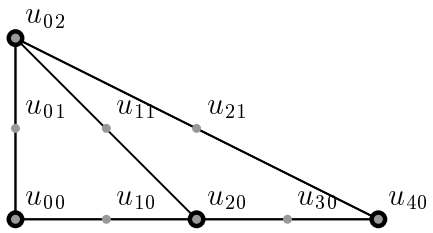
2	1				
1	6	0	2		
0	0	0	0	0	2
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
2 \cdot u_{21} + 2 \cdot u_{01} &< 4 \cdot u_{11} \\
2 \cdot u_{21} &< u_{40} + u_{02} \\
2 \cdot u_{40} + u_{01} &< 2 \cdot u_{30} + u_{21} \\
2 \cdot u_{40} + 2 \cdot u_{01} &< 2 \cdot u_{20} + 2 \cdot u_{21} \\
2 \cdot u_{40} + 3 \cdot u_{01} &< 2 \cdot u_{10} + 3 \cdot u_{21} \\
2 \cdot u_{40} + 4 \cdot u_{01} &< 2 \cdot u_{00} + 4 \cdot u_{21}
\end{aligned}$$

29. Vertex: $u_{40}u_{20}^4u_{00}u_{02}^6 \equiv$ (0,1,0,4,0,1,0,0,6)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $u_{40} + 4 \cdot u_{20} + u_{00} + 6 \cdot u_{02}$

2	2				
1	0	-4	0		
0	1	0	0	0	1
$c_T(i,j)$	0	1	2	3	4

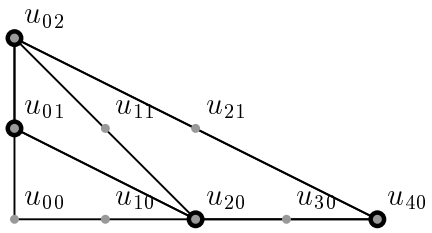
2	5				
1	0	0	0		
0	1	0	4	0	1
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned} 4 \cdot u_{20} + 4 \cdot u_{02} &< 8 \cdot u_{11} \\ 4 \cdot u_{20} &< 2 \cdot u_{40} + 2 \cdot u_{00} \\ u_{00} + u_{02} &< 2 \cdot u_{01} \\ u_{20} + u_{00} &< 2 \cdot u_{10} \\ u_{40} + u_{20} &< 2 \cdot u_{30} \\ u_{40} + u_{02} &< 2 \cdot u_{21} \end{aligned}$$

30. Vertex: $u_{40}u_{20}^4u_{01}^2u_{02}^5 \equiv$ (0,1,0,4,0,0,0,2,5)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $u_{40} + 4 \cdot u_{20} + 2 \cdot u_{01} + 5 \cdot u_{02}$

2	2				
1	2	-6	0		
0	0	0	1	0	1
$c_T(i,j)$	0	1	2	3	4

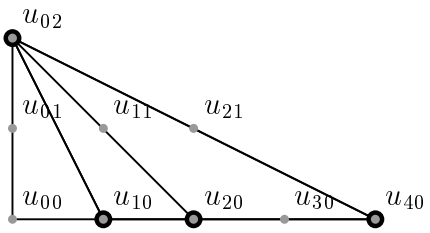
2	4				
1	2	0	0		
0	0	0	4	0	1
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
3 \cdot u_{20} + 3 \cdot u_{02} &< 6 \cdot u_{11} \\
4 \cdot u_{20} + 2 \cdot u_{02} &< 2 \cdot u_{40} + 4 \cdot u_{01} \\
2 \cdot u_{01} &< u_{00} + u_{02} \\
u_{20} + 2 \cdot u_{01} &< 2 \cdot u_{10} + u_{02} \\
u_{40} + u_{20} &< 2 \cdot u_{30} \\
u_{40} + u_{02} &< 2 \cdot u_{21}
\end{aligned}$$

31. Vertex: $u_{40}u_{20}^3u_{10}^2u_{02}^6 \equiv$ (0,1,0,3,2,0,0,0,6)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $u_{40} + 3 \cdot u_{20} + 2 \cdot u_{10} + 6 \cdot u_{02}$

2	3				
1	0	-6	0		
0	0	2	0	0	1
$c_T(i,j)$	0	1	2	3	4

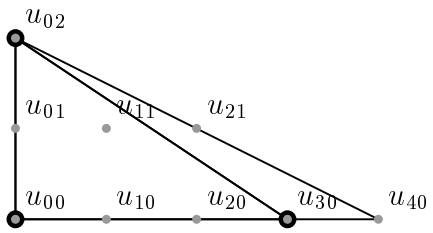
2	5				
1	0	0	0		
0	0	2	3	0	1
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned} 3 \cdot u_{20} + 3 \cdot u_{02} &< 6 \cdot u_{11} \\ 3 \cdot u_{20} &< u_{40} + 2 \cdot u_{10} \\ 2 \cdot u_{10} &< u_{20} + u_{00} \\ 2 \cdot u_{10} + u_{02} &< u_{20} + 2 \cdot u_{01} \\ u_{40} + u_{20} &< 2 \cdot u_{30} \\ u_{40} + u_{02} &< 2 \cdot u_{21} \end{aligned}$$

32. Vertex: $u_{30}^4 u_{00}^2 u_{02}^6 \equiv$ (0,0,4,0,0,2,0,0,6)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $4 \cdot u_{30} + 2 \cdot u_{00} + 6 \cdot u_{02}$

2	3				
1	0	-6	0		
0	1	0	0	2	0
$c_T(i,j)$	0	1	2	3	4

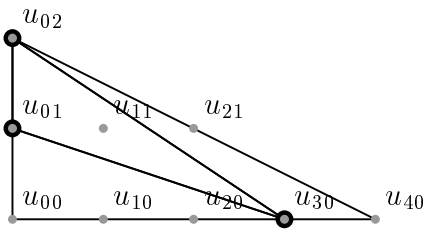
2	5				
1	0	0	0		
0	2	0	0	4	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned} 2 \cdot u_{30} + u_{00} + 3 \cdot u_{02} &< 6 \cdot u_{11} \\ 4 \cdot u_{30} &< 3 \cdot u_{40} + u_{00} \\ 2 \cdot u_{00} + 2 \cdot u_{02} &< 4 \cdot u_{01} \\ u_{30} + 2 \cdot u_{00} &< 3 \cdot u_{10} \\ 2 \cdot u_{30} + u_{00} &< 3 \cdot u_{20} \\ 4 \cdot u_{30} + 3 \cdot u_{02} &< u_{00} + 6 \cdot u_{21} \end{aligned}$$

33. Vertex: $u_{30}^4 u_{01}^4 u_{02}^4 \equiv$ (0,0,4,0,0,0,0,4,4)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $4 \cdot u_{30} + 4 \cdot u_{01} + 4 \cdot u_{02}$

2	3				
1	3	-9	0		
0	0	0	0	3	0
$c_T(i,j)$	0	1	2	3	4

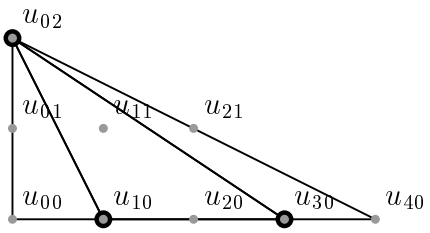
2	3				
1	4	0	0		
0	0	0	0	4	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{30} + u_{01} + u_{02} &< 3 \cdot u_{11} \\
4 \cdot u_{30} + u_{02} &< 3 \cdot u_{40} + 2 \cdot u_{01} \\
4 \cdot u_{01} &< 2 \cdot u_{00} + 2 \cdot u_{02} \\
u_{30} + 4 \cdot u_{01} &< 3 \cdot u_{10} + 2 \cdot u_{02} \\
2 \cdot u_{30} + 2 \cdot u_{01} &< 3 \cdot u_{20} + u_{02} \\
2 \cdot u_{30} + 2 \cdot u_{02} &< 3 \cdot u_{21} + u_{01}
\end{aligned}$$

34. Vertex: $u_{30}^3 u_{10}^3 u_{02}^6 \equiv$ (0,0,3,0,3,0,0,0,6)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $3 \cdot u_{30} + 3 \cdot u_{10} + 6 \cdot u_{02}$

2	4				
1	0	-8	0		
0	0	2	0	2	0
$c_T(i,j)$	0	1	2	3	4

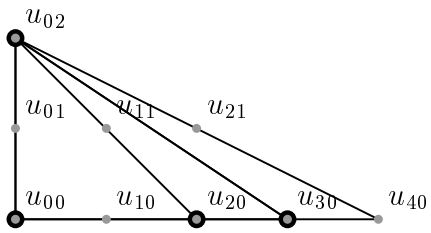
2	5				
1	0	0	0		
0	0	3	0	3	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{30} + u_{10} + 2 \cdot u_{02} &< 4 \cdot u_{11} \\
3 \cdot u_{30} &< 2 \cdot u_{40} + u_{10} \\
3 \cdot u_{10} &< u_{30} + 2 \cdot u_{00} \\
3 \cdot u_{10} + 2 \cdot u_{02} &< u_{30} + 4 \cdot u_{01} \\
u_{30} + u_{10} &< 2 \cdot u_{20} \\
3 \cdot u_{30} + 2 \cdot u_{02} &< u_{10} + 4 \cdot u_{21}
\end{aligned}$$

35. Vertex: $u_{30}^2 u_{20}^3 u_{00} u_{02}^6 \equiv$ (0,0,2,3,0,1,0,0,6)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $2 \cdot u_{30} + 3 \cdot u_{20} + u_{00} + 6 \cdot u_{02}$

2	3				
1	0	-6	0		
0	1	0	0	2	0
$c_T(i,j)$	0	1	2	3	4

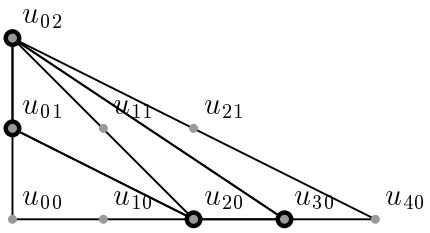
2	5				
1	0	0	0		
0	1	0	3	2	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
3 \cdot u_{20} + 3 \cdot u_{02} &< 6 \cdot u_{11} \\
2 \cdot u_{30} &< u_{40} + u_{20} \\
3 \cdot u_{20} &< 2 \cdot u_{30} + u_{00} \\
u_{00} + u_{02} &< 2 \cdot u_{01} \\
u_{20} + u_{00} &< 2 \cdot u_{10} \\
2 \cdot u_{30} + u_{02} &< u_{20} + 2 \cdot u_{21}
\end{aligned}$$

36. Vertex: $u_{30}^2 u_{20}^3 u_{01}^2 u_{02}^5 \equiv$ (0,0,2,3,0,0,0,2,5)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $2 \cdot u_{30} + 3 \cdot u_{20} + 2 \cdot u_{01} + 5 \cdot u_{02}$

2	3				
1	2	-8	0		
0	0	0	1	2	0
$c_T(i,j)$	0	1	2	3	4

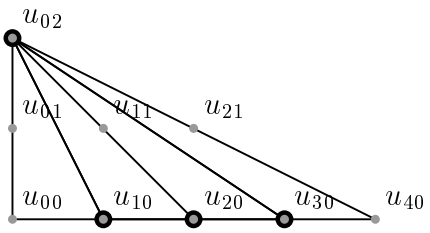
2	4				
1	2	0	0		
0	0	0	3	2	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned} 2 \cdot u_{20} + 2 \cdot u_{02} &< 4 \cdot u_{11} \\ 2 \cdot u_{30} &< u_{40} + u_{20} \\ 3 \cdot u_{20} + u_{02} &< 2 \cdot u_{30} + 2 \cdot u_{01} \\ 2 \cdot u_{01} &< u_{00} + u_{02} \\ u_{20} + 2 \cdot u_{01} &< 2 \cdot u_{10} + u_{02} \\ 2 \cdot u_{30} + u_{02} &< u_{20} + 2 \cdot u_{21} \end{aligned}$$

37. Vertex: $u_{30}^2u_{20}^2u_{10}^2u_{02}^6 \equiv$ (0,0,2,2,2,0,0,0,6)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $2 \cdot u_{30} + 2 \cdot u_{20} + 2 \cdot u_{10} + 6 \cdot u_{02}$

2	4				
1	0	-8	0		
0	0	2	0	2	0
$c_T(i,j)$	0	1	2	3	4

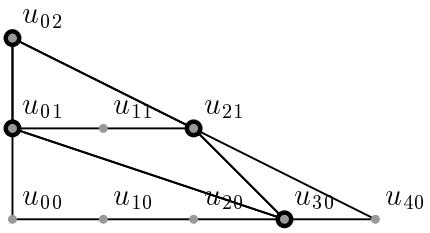
2	5				
1	0	0	0		
0	0	2	2	2	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned} 2 \cdot u_{20} + 2 \cdot u_{02} &< 4 \cdot u_{11} \\ 2 \cdot u_{30} &< u_{40} + u_{20} \\ 2 \cdot u_{20} &< u_{30} + u_{10} \\ 2 \cdot u_{10} &< u_{20} + u_{00} \\ 2 \cdot u_{10} + u_{02} &< u_{20} + 2 \cdot u_{01} \\ 2 \cdot u_{30} + u_{02} &< u_{20} + 2 \cdot u_{21} \end{aligned}$$

38. Vertex: $u_{30}^2 u_{21}^3 u_{01}^5 u_{02}^2 \equiv$ (0,0,2,0,0,0,3,5,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $2 \cdot u_{30} + 3 \cdot u_{21} + 5 \cdot u_{01} + 2 \cdot u_{02}$

2	2				
1	3	-8	1		
0	0	0	0	2	0
$c_T(i,j)$	0	1	2	3	4

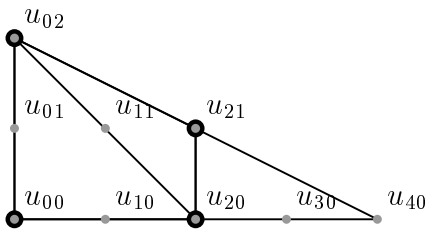
2	1				
1	5	0	3		
0	0	0	0	2	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
2 \cdot u_{21} + 2 \cdot u_{01} &< 4 \cdot u_{11} \\
2 \cdot u_{30} + u_{21} &< 2 \cdot u_{40} + u_{01} \\
3 \cdot u_{21} + u_{01} &< 2 \cdot u_{30} + 2 \cdot u_{02} \\
2 \cdot u_{30} + u_{01} &< 2 \cdot u_{20} + u_{21} \\
2 \cdot u_{30} + 2 \cdot u_{01} &< 2 \cdot u_{10} + 2 \cdot u_{21} \\
2 \cdot u_{30} + 3 \cdot u_{01} &< 2 \cdot u_{00} + 3 \cdot u_{21}
\end{aligned}$$

39. Vertex: $u_{20}^4 u_{00} u_{21}^2 u_{02}^5 \equiv$ (0,0,0,4,0,1,2,0,5)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $4 \cdot u_{20} + u_{00} + 2 \cdot u_{21} + 5 \cdot u_{02}$

2	2				
1	0	-6	2		
0	1	0	1	0	0
$c_T(i,j)$	0	1	2	3	4

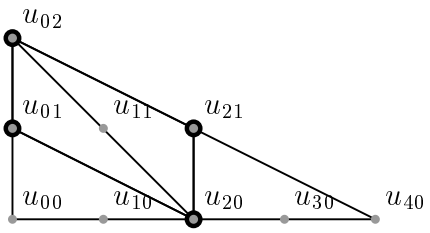
2	4				
1	0	0	2		
0	1	0	4	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
3 \cdot u_{20} + 3 \cdot u_{02} &< 6 \cdot u_{11} \\
2 \cdot u_{21} &< u_{40} + u_{02} \\
u_{20} + 2 \cdot u_{21} &< 2 \cdot u_{30} + u_{02} \\
u_{00} + u_{02} &< 2 \cdot u_{01} \\
u_{20} + u_{00} &< 2 \cdot u_{10} \\
4 \cdot u_{20} + 2 \cdot u_{02} &< 2 \cdot u_{00} + 4 \cdot u_{21}
\end{aligned}$$

40. Vertex: $u_{20}^4 u_{21}^2 u_{01}^2 u_{02}^4 \equiv$ (0,0,0,4,0,0,2,2,4)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $4 \cdot u_{20} + 2 \cdot u_{21} + 2 \cdot u_{01} + 4 \cdot u_{02}$

2	2				
1	2	-8	2		
0	0	0	2	0	0
$c_T(i,j)$	0	1	2	3	4

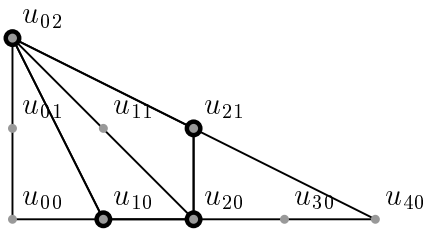
2	3				
1	2	0	2		
0	0	0	4	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
2 \cdot u_{20} + 2 \cdot u_{02} &< 4 \cdot u_{11} \\
2 \cdot u_{21} &< u_{40} + u_{02} \\
u_{20} + 2 \cdot u_{21} &< 2 \cdot u_{30} + u_{02} \\
2 \cdot u_{01} &< u_{00} + u_{02} \\
u_{20} + 2 \cdot u_{01} &< 2 \cdot u_{10} + u_{02} \\
2 \cdot u_{20} + 2 \cdot u_{02} &< 2 \cdot u_{21} + 2 \cdot u_{01}
\end{aligned}$$

41. Vertex: $u_{20}^3 u_{10}^2 u_{21}^2 u_{02}^5 \equiv$ (0,0,0,3,2,0,2,0,5)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $3 \cdot u_{20} + 2 \cdot u_{10} + 2 \cdot u_{21} + 5 \cdot u_{02}$

2	3				
1	0	-8	2		
0	0	2	1	0	0
$c_T(i,j)$	0	1	2	3	4

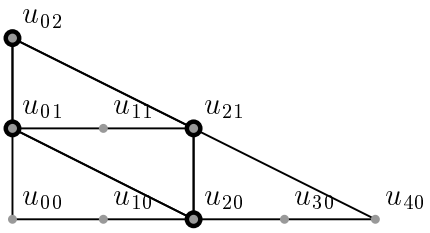
2	4				
1	0	0	2		
0	0	2	3	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
2 \cdot u_{20} + 2 \cdot u_{02} &< 4 \cdot u_{11} \\
2 \cdot u_{21} &< u_{40} + u_{02} \\
u_{20} + 2 \cdot u_{21} &< 2 \cdot u_{30} + u_{02} \\
2 \cdot u_{10} &< u_{20} + u_{00} \\
2 \cdot u_{10} + u_{02} &< u_{20} + 2 \cdot u_{01} \\
3 \cdot u_{20} + u_{02} &< 2 \cdot u_{10} + 2 \cdot u_{21}
\end{aligned}$$

42. Vertex: $u_{20}^2u_{21}^4u_{01}^4u_{02}^2 \equiv$ (0,0,0,2,0,0,4,4,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $2 \cdot u_{20} + 4 \cdot u_{21} + 4 \cdot u_{01} + 2 \cdot u_{02}$

2	2				
1	2	-8	2		
0	0	0	2	0	0
$c_T(i,j)$	0	1	2	3	4

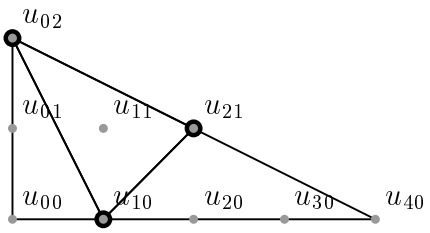
2	1				
1	4	0	4		
0	0	0	2	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
2 \cdot u_{21} + 2 \cdot u_{01} &< 4 \cdot u_{11} \\
2 \cdot u_{20} + 2 \cdot u_{21} &< 2 \cdot u_{40} + 2 \cdot u_{01} \\
2 \cdot u_{20} + u_{21} &< 2 \cdot u_{30} + u_{01} \\
2 \cdot u_{21} + 2 \cdot u_{01} &< 2 \cdot u_{20} + 2 \cdot u_{02} \\
2 \cdot u_{20} + u_{01} &< 2 \cdot u_{10} + u_{21} \\
2 \cdot u_{20} + 2 \cdot u_{01} &< 2 \cdot u_{00} + 2 \cdot u_{21}
\end{aligned}$$

43. Vertex: $u_{10}^4 u_{21}^4 u_{02}^4 \equiv$ (0,0,0,0,4,0,4,0,4)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $4 \cdot u_{10} + 4 \cdot u_{21} + 4 \cdot u_{02}$

2	3				
1	0	-9	3		
0	0	3	0	0	0
$c_T(i,j)$	0	1	2	3	4

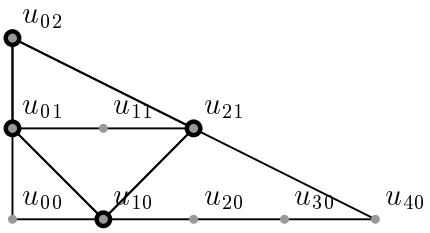
2	3				
1	0	0	4		
0	0	4	0	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
u_{10} + u_{21} + u_{02} &< 3 \cdot u_{11} \\
4 \cdot u_{21} &< 2 \cdot u_{40} + 2 \cdot u_{02} \\
u_{10} + 4 \cdot u_{21} &< 3 \cdot u_{30} + 2 \cdot u_{02} \\
2 \cdot u_{10} + 2 \cdot u_{21} &< 3 \cdot u_{20} + u_{02} \\
2 \cdot u_{10} + 2 \cdot u_{02} &< u_{21} + 3 \cdot u_{01} \\
4 \cdot u_{10} + u_{02} &< 3 \cdot u_{00} + 2 \cdot u_{21}
\end{aligned}$$

44. Vertex: $u_{10}^2 u_{21}^5 u_{01}^3 u_{02}^2 \equiv$ (0,0,0,0,2,0,5,3,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $2 \cdot u_{10} + 5 \cdot u_{21} + 3 \cdot u_{01} + 2 \cdot u_{02}$

2	2				
1	1	-8	3		
0	0	2	0	0	0
$c_T(i,j)$	0	1	2	3	4

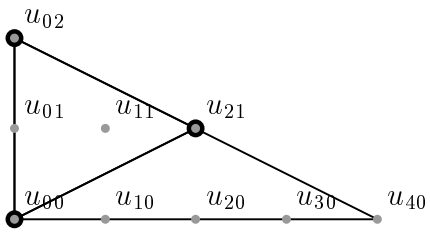
2	1				
1	3	0	5		
0	0	2	0	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
2 \cdot u_{21} + 2 \cdot u_{01} &< 4 \cdot u_{11} \\
2 \cdot u_{10} + 3 \cdot u_{21} &< 2 \cdot u_{40} + 3 \cdot u_{01} \\
2 \cdot u_{10} + 2 \cdot u_{21} &< 2 \cdot u_{30} + 2 \cdot u_{01} \\
2 \cdot u_{10} + u_{21} &< 2 \cdot u_{20} + u_{01} \\
u_{21} + 3 \cdot u_{01} &< 2 \cdot u_{10} + 2 \cdot u_{02} \\
2 \cdot u_{10} + u_{01} &< 2 \cdot u_{00} + u_{21}
\end{aligned}$$

45. Vertex: $u_{00}^3 u_{21}^6 u_{02}^3 \equiv$ (0,0,0,0,0,3,6,0,3)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $3 \cdot u_{00} + 6 \cdot u_{21} + 3 \cdot u_{02}$

2	2				
1	0	-8	4		
0	2	0	0	0	0
$c_T(i,j)$	0	1	2	3	4

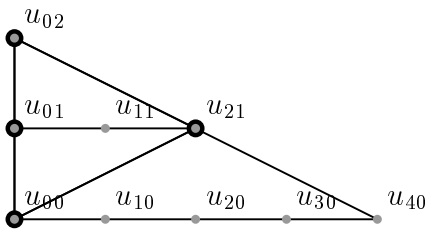
2	2				
1	0	0	6		
0	3	0	0	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned} u_{00} + 2 \cdot u_{21} + u_{02} &< 4 \cdot u_{11} \\ 6 \cdot u_{21} &< 3 \cdot u_{40} + 3 \cdot u_{02} \\ u_{00} + 6 \cdot u_{21} &< 4 \cdot u_{30} + 3 \cdot u_{02} \\ 2 \cdot u_{00} + 4 \cdot u_{21} &< 4 \cdot u_{20} + 2 \cdot u_{02} \\ 3 \cdot u_{00} + 2 \cdot u_{21} &< 4 \cdot u_{10} + u_{02} \\ u_{00} + u_{02} &< 2 \cdot u_{01} \end{aligned}$$

46. Vertex: $u_{00}^2u_{21}^6u_{01}^2u_{02}^2 \equiv$ (0,0,0,0,0,2,6,2,2)

The coarsest Newton subdivision of this cone in the Groebner fan is:



The negative of the cycle length is: 0

The generic valuation of Δ is: $2 \cdot u_{00} + 6 \cdot u_{21} + 2 \cdot u_{01} + 2 \cdot u_{02}$

2	2				
1	0	-8	4		
0	2	0	0	0	0
$c_T(i,j)$	0	1	2	3	4

2	1				
1	2	0	6		
0	2	0	0	0	0
$\eta_T(i,j)$	0	1	2	3	4

The inequalities describing the cone of the Groebner fan of Δ dual to the above vertex:

$$\begin{aligned}
2 \cdot u_{21} + 2 \cdot u_{01} &< 4 \cdot u_{11} \\
2 \cdot u_{00} + 4 \cdot u_{21} &< 2 \cdot u_{40} + 4 \cdot u_{01} \\
2 \cdot u_{00} + 3 \cdot u_{21} &< 2 \cdot u_{30} + 3 \cdot u_{01} \\
2 \cdot u_{00} + 2 \cdot u_{21} &< 2 \cdot u_{20} + 2 \cdot u_{01} \\
2 \cdot u_{00} + u_{21} &< 2 \cdot u_{10} + u_{01} \\
2 \cdot u_{01} &< u_{00} + u_{02}
\end{aligned}$$