Eberhard Karls Universität Tübingen Sommersemester 2019
Fachbereich Mathematik
PD Dr. Jörg Zintl

Algebraische Kurven

Übungsaufgaben zum 2. Tutorium am 08.05.2019

Aufgabe 7.
Verify Bézout's theorem in the following examples by explicit computation of $i_P(C, D)$ for all $P \in \mathbb{P}^2$, where

a) $C := V(XY - Z^2)$ and $D := V(XZ - Y^2)$;

b) $C := V(X^3 - Y^2 Z)$ and $D := V(Y^2 Z - X^3 - X^2 Z)$.

Draw pictures of the curves in suitable neighbourhoods of the intersection points.

Aufgabe 8.
Let $C, D \in \mathbb{P}^2_{\mathbb{C}}$ be two projective cubic curves, defined by minimal homogeneous polynomials $F, G \in \mathbb{C}[X, Y, Z]$. Suppose that C and D meet in exactly nine distinct points $P_1, \ldots, P_9$.

a) Show that no 4 of the points $P_1, \ldots, P_9$ lie on a line, and no 7 of them lie on a conic.

b) Show that there exists a unique conic K containing $P_1, \ldots, P_5$.

Aufgabe 9. (Keine Abgabe, Präsenzübung)
Let $C, D \in \mathbb{P}^2_{\mathbb{C}}$ be two projective curves, both of degree d, with minimal polynomials $F, G \in \mathbb{C}[X, Y, Z]$. Suppose that C and D intersect in exactly d^2 distinct points.

Let E be an irreducible curve of degree $m < d$, and let $Q = (x_0 : y_0 : z_0) \in E$, with $Q \notin C \cap D$. Assume that E contains exactly dm distinct points of $C \cap D$.

a) Show that $E \subseteq V(\lambda F + \mu G)$, where $\lambda := G(x_0, y_0, z_0)$ and $\mu := -F(x_0, y_0, z_0)$.

b) Show that the remaining $d(d - m)$ points of the intersection $C \cap D$ lie on a curve B of degree at most $d - m$.

c) Consider a circle S (or more generally an irreducible conic). Draw inside the circle a hexagon, such that all of its vertices lie on S. Each side of the hexagon determines a unique line containing it. For each pair of lines corresponding to opposite sides there is a unique point of intersection, which is not contained in S. Thus the three pairs of opposing sides determine three points.

Show that these three points lie on one line.

Abgabe der Lösungen zu Aufgaben 7 und 8 am 02.05.2019 in der Vorlesung.