**Eberhard Karls Universität Tübingen**
# Fachbereich Mathematik
PD Dr. Jörg Zintl

Sommersemester 2019

# Algebraische Kurven

## Übungsaufgaben zum 3. Tutorium am 15.05.2019

**Aufgabe 10. (Keine Abgabe, Präsenzübung)**
Construct an example of an irreducible quartic $C \subset \mathbb{P}^2$ with exactly 3 singular points. Discuss the numbers of singularities in all cases, where $C$ is not irreducible.

**Aufgabe 11.**
Let $C \subset \mathbb{P}^2$ be a smooth projective plane curve. Show that the ring of regular functions at $P$

$$\mathcal{O}_{C,P} := \left\{ f \in k(C) : \exists\, g, h \in k[C] \text{ with } h(P) \neq 0 \text{ such that } f = \frac{g}{h} \right\}$$

is a local ring, and a principal ideal domain.

**Aufgabe 12. (Keine Abgabe, Präsenzübung)**
Let $C \subset \mathbb{P}^2$ be a smooth projective curve. Let $G \in \mathbb{C}[X, Y, Z]$ be an irreducible homogeneous polynomial of degree $d$, and let $D := V(G)$. Let $P \in C$, and let $\ell \in \mathbb{C}[X, Y, Z]$ be a linear polynomial such that $P \notin V(\ell)$. Show that for the order of $\frac{G}{\ell^d} \in k(C)$ holds

$$\nu_P\left(\frac{G}{\ell^d}\right) = i_P(C, D).$$

(You may assume $P = (0 : 0 : 1)$ and $\ell(X, Y, Z) = Z$.)

**Aufgabe 13.**
Consider the smooth projective plane curves $L := V(Z)$ and $C := V(XY - Z^2)$ in $\mathbb{P}^2$, together with the morphism $\varphi : L \to C$ given by

$$\varphi := \left(\frac{X}{Y} : \frac{Y}{X} : 1\right).$$

Show that $\varphi$ is in isomorphism by constructing an inverse morphism $\psi : C \to L$.

**Abgabe der Lösungen zu Aufgaben 11 und 13 am 15.05.2019 in der Übung.**