Eberhard Karls Universität Tübingen Sommersemester 2019
Fachbereich Mathematik
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Algebraische Kurven

Übungsaufgaben zum 6. Tutorium am 19.06.2019

Aufgabe 21.
Let $\varphi : C \to C'$ and $\psi : C' \to C''$ be morphisms between smooth projective plane curves. Show that for the respective ramification divisors holds

$$R_{\psi\circ\varphi} \;=\; \varphi^* R_\psi + R_\varphi .$$

Aufgabe 22.
Let R be an integral domain, and a $\mathbb{C}$-algebra. Let $K := \mathrm{Quot}(R)$ be its quotient field. Let M be a vector space over K.

Show that any derivation $D : R \to M$ extends uniquely to a derivation $\overline{D} : K \to M$.

Aufgabe 23. (Keine Abgabe, Präsenzübung)
a) Let R, S be $\mathbb{C}$-algebras. Consider the $\mathbb{C}$-algebra $T := R \otimes_{\mathbb{C}} S$, where the multiplication is given by $(r \otimes s) \cdot (r' \otimes s') := rr' \otimes ss'$, for $r, r' \in R$ and $s, s' \in S$. Show that there is an isomorphism of T-modules

$$\Omega_T \;\cong\; T \otimes_R \Omega_R \;\oplus\; T \otimes_S \Omega_S .$$

Hint: use the identifications $T \otimes_R \Omega_R = S \otimes_{\mathbb{C}} \Omega_R$ and $T \otimes_R \Omega_S = R \otimes_{\mathbb{C}} \Omega_S$, and consider the map

$$\begin{array}{rccc} \Delta : & R \times S & \to & S \otimes_{\mathbb{C}} \Omega_R \oplus R \otimes_{\mathbb{C}} \Omega_S \\ & (r,s) & \mapsto & s \otimes dr + r \otimes ds. \end{array}$$

b) For the ring of polynomials in n variables, describe $\Omega_{\mathbb{C}[X_1,\dots,X_n]}$.

Aufgabe 24.
Let $C \subset \mathbb{P}^2$ be a smooth projective plane curve of degree $d = 3$. Let $\mathcal{O} \in C$ be an inflection point, and let "$\oplus$" be the Abelian group structure on C from exercise 20.

a) Let $\mathrm{Cl}_0(C)$ denote the subgroup consisting of those divisor classes $D \in \mathrm{Cl}(C)$ with degree $\deg(D) = 0$. Construct a bijection

$$\mathrm{Cl}_0(C) \;\cong\; \{P - \mathcal{O} \in \mathrm{Cl}(C) :\; P \in C\}.$$

b) Show that there exists an isomorphism of groups

$$(C, \oplus) \;\cong\; (\mathrm{Cl}_0(C), +).$$

Abgabe der Lösungen zu Aufgaben 21, 22, 24 am 19.06.2019 in der Übung.