Eberhard Karls Universität Tübingen Sommersemester 2019

Fachbereich Mathematik

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Algebraische Kurven

Übungsaufgaben zum 8. Tutorium am 03.07.2019

Aufgabe 27.

a) Let $C \subset \mathbb{P}^2$ be a smooth projective plane curve, given by a non-constant homogeneous minimal polynomial $F \in \mathbb{C}[X,Y,Z]$. Let $P \in C$ be a point. Show that the unique tangent to C in P is given by

$$L := V\left(\frac{\partial F}{\partial X}(P) \cdot X + \frac{\partial F}{\partial Y}(P) \cdot Y + \frac{\partial F}{\partial Z}(P) \cdot Z\right).$$

b) Construct the dual curve $C^* \subset (\mathbb{P}^2)^*$ of the curve $C := V(X^2 - YZ) \subset \mathbb{P}^2$.

Aufgabe 28. (Keine Abgabe, Präsenzübung)

Let $C \subset \mathbb{P}^2$ be a smooth projective plane curve of degree d. Let $P_0 \in \mathbb{P}^2$ be a point, and let $L \subset \mathbb{P}^2$ be a line with $L \neq C$ and $P_0 \notin L$. The *projection onto* L *with center* P_0 is constructed as follows. For any point $P \in C$, with $P \neq P_0$, let $L_P \subset \mathbb{P}^2$ denote the unique line through P_0 and P. If $P = P_0$, then L_{P_0} shall be the unique tangent to C in P_0. Let $\pi(P)$ denote the unique intersection point of L_P and L.

Show that $\pi : C \to L$ is a morphism, and compute the degree of π in the case $P_0 \notin C$.

You may assume $P_0 = (0:1:0)$ and $L = V(Y)$.

Aufgabe 29. (Keine Abgabe, Präsenzübung)

Let $C \subset \mathbb{P}^2$ be a smooth projective plane curve of degree d. Use arguments similar to those in the proof of Satz (4.32) to explain why an arbitrary point $P \in \mathbb{P}^2 \setminus C$ lies on at most $d(d-1)$ tangents of C.

Aufgabe 30.

Compute the topological genus of a complex torus by constructing a triangulation.

Abgabe der Lösungen zu Aufgaben 27 und 30 am 03.07.2019 in der Übung.