Prof. Dr. Christoph Bohle Tübingen SS 2014

Integrable Systeme : Blatt 1

Dr. Aaron Gerding 10. April 2014

Diese Aufgaben sind schriftlich auszuarbeiten und bis zum 21. April abzugeben. Für jede Aufgabe gibt es 4 Punkte.

Aufgabe 1. Verify the general solution formula for the wave equation

$$\partial_t^2 u - \partial_x^2 u = 0$$

on $\mathbb{R}^2$ as given in the lecture. (Hint: transform to lightcone coordinates $\xi = x + t$ and $\nu = x - t$.)

Aufgabe 2. Verify directly that the 1–soliton solutions given in the lecture solve the KdV–equation. Show that they are the only traveling wave solutions of the KdV–equation for which the derivatives up to order two vanish when $x \to \pm\infty$. (Hint: derive the ODE characterizing f's such that $u(x,t) = f(x - ct)$ solves the KdV–equation. Integrate once, multiply by f' and integrate again.)

Aufgabe 3. Determine the homogeneous differential polynomials $\hat{K}(u)$ of weight 7 that give rise to symmetries of the KdV–equation.

Aufgabe 4. Determine all linear differential operators of the form

$$A = \partial_x^n + \sum_{k=0}^{n-1} a_k(x)\partial_x^k$$

with $n = 1,..., 5$ such that the operator $[A, L]$ with $L = \partial_x^2 + u(x)$ has order zero.