Prof. Dr. Christoph Bohle | Tübingen SS 2014

Integrable Systeme : Blatt 6

Dr. Aaron Gerding | 15. Mai 2014

Diese Aufgaben sind schriftlich auszuarbeiten und bis zum 26. Mai abzugeben. Für jede Aufgabe gibt es 4 Punkte.

Aufgabe 1. Show (without using W as in the lecture):

i) For a given micro differential operator $L = \partial + \sum_{i=1}^{\infty} u_i \partial^{-i}$ there is a Baker function $\psi = e^{\lambda x}(1 + \sum_{i=1}^{\infty} w_i \lambda^{-i})$ such that

$$L\psi = \lambda\psi.$$

The Baker function is unique up to multiplication by a constant coefficient series $(1 + \sum_{i=1}^{\infty} c_i \lambda^{-i})$.

ii) Conversely, for a given Baker function $\psi = e^{\lambda x}(1 + \sum_{i=1}^{\infty} w_i \lambda^{-i})$ there is a unique L of the above form with $L\psi = \lambda\psi$.

Aufgabe 2. Show that $\psi = W e^{\xi(\underline{t},\lambda)}$ solves $L\psi = \lambda\psi$ and $\frac{\partial}{\partial_{t_k}}\psi = B_k\psi$, $k \geq 1$ if and only if W solves the Sato–Wilson equation.

Aufgabe 3. Show that $Q\psi = 0$ implies $Q = 0$, where Q denotes a micro differential operator and ψ a Baker function.