Prof. Dr. Christoph Bohle Tübingen SS 2014

Integrable Systeme : Blatt 9

Dr. Aaron Gerding 5. Juni 2014

Diese Aufgaben sind schriftlich auszuarbeiten und bis zum 23. Juni abzugeben. Für jede Aufgabe gibt es 4 Punkte.

Aufgabe 1. Let ξ be a frame representing a point of Sato's universal Grassmannian UGr.

a) Show that there is a unique minimal Young diagramm Y such that the corresponding Plücker coordinate ξ_Y is non–zero.
b) Show that ξ represents a point in the big cell if and only if $\xi_\emptyset \neq 0$.

Aufgabe 2. Let $H = H_- \oplus H_+$ be a vector space. Then the (index zero part of the) Grassmannian $Gr_0(H)$ is defined as the subspaces $U \subset H$ such that the projection pr_{H_-} to H_- is Fredholm of index zero, i.e., has finite dimensional kernel and cokernel of the same dimension. Show that:

a) Sato's universal Grassmannian UGr can be identified with $Gr_0(H)$ for $H = \mathbb{C}((y))$ with $H_- = y^{-1}\mathbb{C}[y^{-1}]$ and $H_+ = \mathbb{C}[[y]]$.
b) The big cell $UGr^\emptyset$ corresponds to points U for which pr_{H_-} is a bijection. The later is equivalent to $H = U \oplus H_+$. Such U's are precisely the graphs of operators $H_- \to H_+$.

Aufgabe 3. Show that a $\mathcal{D}$–module $\mathcal{J} \subset \mathcal{E}$ satisfies $\mathcal{D} \oplus \mathcal{E}_- = \mathcal{E}$ if and only if it is of the form $\mathcal{J} = \mathcal{D}W$ for some monic micro differential operator W of degree 0.

Aufgabe 4. Let $\mathcal{J}(t) = \mathcal{D}W(t)$ be a family of $\mathcal{D}$–modules as above. Show that the following are equivalent:

i) W satisfies the n^{th} Sato-Wilson equation

$$\frac{d}{dt}W = -(W \circ \partial^n \circ W^{-1})_- W.$$

ii) $\frac{d}{dt}W + W \circ \partial^n \in \mathcal{J}(t)$ for all t
iii) $\frac{d}{dt}\tilde{W} + \tilde{W} \circ \partial^n \in \mathcal{J}(t)$ for all t and all families $\tilde{W}(t)$ with $\tilde{W}(t) \in \mathcal{J}(t)$