Integrable Systeme : Blatt 11

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Diese Aufgaben sind schriftlich auszuarbeiten und bis zum 7. Juli abzugeben. Für jede Aufgabe gibt es 4 Punkte.

Aufgabe 1. Denote by $\mathcal{A}$ the associative algebra over $\mathbb{C}$ generated by ψ^* and ψ under the relations $\psi^2 = (\psi^*)^2 = 0$ and $\psi\psi^* + \psi^*\psi = 1$. Compute the dimension of $\mathcal{A}$. Compute the matrices representing the action of ψ and ψ^* on the Fock space $\mathcal{F} = \mathcal{A}/\mathcal{A}\psi$ with respect to the basis $|vac\rangle$ and $\psi^*|vac\rangle$.

Aufgabe 2. Compute how the vector

$$\psi_{m_1}...\psi_{m_r}\psi^*_{n_1}...\psi^*_{n_s}|vac\rangle,$$

$m_1 < ... < m_r < 0$, $n_1 < ... < n_s < 0$, is written with respect to the second ("semi–infinite wedge" or "Maya diagram") definition of fermionic Fock space given in the lecture (including the correct sign).

Aufgabe 3. Show that for given $l \in \mathbb{Z}$ the vector

$$|l\rangle = \begin{cases} \psi^*_{l+1/2}, ..., \psi^*_{-1/2}|vac\rangle & l < 0 \\ |vac\rangle & l = 0 \\ \psi_{-l+1/2}...\psi_{-1/2}|vac\rangle & l > 0 \end{cases}$$

has minimal energy among all vectors of charge l in fermionic Fock space $\mathcal{F}$. Compute its energy. Give analogous minimal energy vectors of the dual fermionic Fock space $\mathcal{F}^*$. (The charge and energy gradings of the free Fermion algebra $\mathcal{A}$ are characterized by the fact that ψ_i has charge $+1$ and energy $-i$ while ψ^*_i has charge -1 and energy $-i$. The charge and energy gradings of the Fock spaces $\mathcal{F}$ and $\mathcal{F}^*$ are defined as follows: for monomial $a \in \mathcal{A}$, the charge and energy of $a|vac\rangle$ is equal to the charge/energy of a, while the charge and energy of $\langle vac|a$ is *minus* the charge/energy of a.)