Improvements and corrections to:

The Solution of the k(GV) Problem

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As usual the symbols a^b and a_b refer to line b on page a from above and below, respectively.

8^{15} : Replace "$\mathrm{Irr}(G/N)$" by "$\mathrm{Irr}(X/N)$".

10^9 : Delete "(some)".

18 : The following argument is missing before Theorem 1.10a:
In the proof for Theorem 1.9c we have seen that

$$|\mathrm{Irr}(X|\theta)| = \frac{|G|}{|X|^2} \sum_{x\in X} \sum_{y \in C_X(Nx)} \theta([x,y]).$$

Now $y \mapsto \theta([x,y])$ is a linear character of $C_X(Nx)$ which is trivial if and only if the class of Nx is good for θ; otherwise $\sum_{y\in C_X(Nx)} \theta([x,y]) = 0$. An obvious application of the Cauchy–Frobenius formula (1.4a) therefore yields the following.

28^4 : Insert the following: Write $\sum_{\varphi\in \mathrm{IBr}(b)} d^y_{\chi\varphi}\varphi(1) = \sum_j \big(\sum_\varphi d^{(j)}_{\chi\varphi}\varphi(1)\big)\varepsilon^j$ with integers $d^{(j)}_{\chi\varphi}$ (for each χ).

35^3 : Delete "and has prime order".

35_7 : A bracket is missing: $T = N_{\mathrm{GL}(V)}(S)$

36^7 : Hence, *using Theorem 1.4b* and the Clifford–Gallagher formula (1.10b), we get the (crude) estimate ...

39_{11} : The congruence should read $\equiv \psi(1)\chi(v^{-1}) \equiv \pm\chi(1) \pmod{\mathfrak{p}}$.

41^5 : Replace $1 < n < m$ by $1 \le n < m$.

41_1 : (3.4b) $\quad C_H(v) = \bigcap_i C_H(v_i) = 1.$

47^7 : Replace $1 < i < n$ by $1 \le i < n$.

47_{16} : "if and only if $r^n = 2^1$, 3^1 or 2^2." (Of course $\mathrm{GL}_2(2) = \mathrm{Sp}_2(2)$.)

51^9 : Replace $(3,1)$ by $(1,3)$.

54_{10} : Let Z be a cyclic central subgroup of X containg $Z(E)$, and let $U = EZ/Z$ (which may be identified with $E/Z(E)$).c

60_{15} : Let V be a coprime symplectic $\mathbb{F}_pG$-module ...

71^4 : Delete "be".

72^9 : The proof of step (3) is incomplete. Argue as follows:

Otherwise choose N so that this is false. Then $N \neq G$ by (1). Let W be the unique irreducible constituent of $\mathrm{Res}_N^G(V)$ (in view of (2)). By assumption $F_0 = \mathrm{End}_{FN}(W)$ is a proper extension field of F. Let $\Gamma = \mathrm{Gal}(F_0|F)$ (a cyclic group), and regard $W_0 = W$ as F_0N-module. Then $F_0 \otimes_F W \cong \bigoplus_{\sigma\in\Gamma} W_0^\sigma$ where the Galois conjugate F_0N-modules W_0^σ are pairwise nonisomorphic (and absolutely irreducible). To every $x \in G$ there exists a unique $\sigma = \sigma_x \in \Gamma$ such that $(W_0x)^\sigma \cong W_0$, and the assignment $x \mapsto \sigma_x$ is a homomorphism making F_0 into a "G-field". The kernel of this homomorphism is a normal subgroup $\widetilde{N}$ of G containing N, and it is the inertia group of each W_0^σ in G. By (1) $F_0 \otimes_F V$ is an (absolutely) irreducible F_0G-module, whose restriction to N contains all the W_0^σ. By Clifford's theorem, for each $\sigma \in \Gamma$, there is a unique (up to isomorphism) irreducible $F_0\widetilde{N}$-module lying above W_0^σ and inducing up to $F_0 \otimes_F V$, and these modules are conjugate under G. We conclude that $\widetilde{N} = N$ and that $\Gamma \cong G/N$. It also follows that $\mathrm{Res}_N^G(V) = W$ is irreducible and that $F_0 \otimes_F V \cong \mathrm{Ind}_N^G(W_0)$. Let $W_0 = W = V$ be as sets in what follows.

Since $\dim_{F_0} W_0 < \dim_F W$, there is $v \in V$ such that the restriction of W_0 to $H_0 = C_N(v)$ contains a faithful F_0H-module U_0 which is self-dual. This is a direct summand of $\mathrm{Res}_{H_0}^N(W_0)$ by Maschke's theorem. Let $H = C_G(v)$. By Mackey decomposition $U = \mathrm{Ind}_{H_0}^H(U_0)$ is a direct summand of $F_0 \otimes_F V$ with $C_H(U) = \bigcap_{h\in H} C_{H_0}(U_0)^h = 1$. Of course U is self-dual. Let χ be the (absolutely irreducible, Frobenius) character of G afforded by V. Since U is a self-dual F_0H-module, for every character θ afforded by some irreducible summand of U, the character θ^* of H, given by $\theta^*(h) = \theta(h^{-1})$ for $h \in H$, is the character of an irreducible summand of U likewise. So $\theta + \theta^*$ is a constituent of $\mathrm{Res}_H^G(\chi)$ if $\theta \neq \theta^*$. For each $\sigma \in \Gamma$ we also have $(\theta^*)^\sigma = (\theta^\sigma)^*$ as a constituent (as $\chi = \chi^\sigma$). Let U_θ be the *irreducible* FH-module affording the trace character $\mathrm{Tr}_{F(\theta)|F}(\theta)$. This U_θ appears in $\mathrm{Res}_H^G(V)$, as well as $U_{\theta^*} = U_\theta^*$ (the dual module). Either $U_\theta \cong U_{\theta^*}$ is self-dual or $U_\theta \oplus U_{\theta^*}$ appears as a (self-dual) FH-submodule of V. Now consider the direct sum of the distinct (nonisomorphic) U_θ, U_{θ^*} obtained in this way. This is a faithful self-dual FH-submodule of V, as desired.

73_{12} : "Let $G_0 = N_G(E_0)$, and let θ, θ_0 be the Brauer characters of E, E_0 afforded by W, W_0, respectively, and let $G(\theta)$, $G_0(\theta_0)$ be the extended representation groups. So $G_0(\theta_0)$ is a central extension of G_0 by a (cyclic) group of order dividing $|G_0|$."

73_7 : Replace $G(\theta_0)$ by $G_0(\theta_0)$.

84^9 : Of course Theorem 4.4 is meant.

91^{17} : ... , because 13 is a Zsigmondy prime divisor of $3^3 - 1$ and so $13 : 3$ cannot be an irreducible subgroup of $\mathrm{Sp}_6(3)$ by Clifford's theorem.

97_3 : Replace $V_V(g)$ by $C_V(\gamma)$ and $2 \cdot 3^2(2r^2)$ by $2 \cdot 2^2(2r^2)$.

100_{13}: Replace E by E_0.

105^{16} : Delete "(hence $r \geq 7$)".

$106_{3,4}$: Replace H_0 by H; the same on 107^2.

112_{16} : Replace $r = 19$ by $r = 13$ (in the table).

113_{11} : Replace (5.1b) by Lemma 5.1b.

114^2 : Correct as follows: "... where χ takes the value -1. Hence $y_V = [z, z^2, z^3, z^4]$ for some element $z \in Z$ of order 5. It follows that the regular E-orbit remains an orbit for $X = E \times \langle z \rangle$, and we may pick the vector v such that $C_X(v) = \langle z^{-1}y \rangle = H$. Since $N_X(H)/H$ has order 2 [Atlas] and $Z(G)/Z(X)$ order 3, we have $H = C_G(v)$ by Lemma 5.1b." (By Proposition 7.3c below, and the discussion on page 132, there are exactly $\frac{30}{6}\mu^6_{\chi,1}(31) = 1$ regular orbit on V for $E \circ Z_6$. This yields at once what we need.)

119^{13} : Replace "four G-orbits" by "one G-orbit".

129_8 : There is a regular G-*orbit* since ...

130_{16} : Insert "by" (Theorem 4.5a).

140_7 : "Let x be an element of L in the Atlas class $3B$. Then x fixes a nonzero vector $v \in V$. From $\chi(2A) = 0$ and $N_L(Fv) \subseteq N_L(\langle x \rangle) \cong 3^2 : 2$ [Atlas] we infer that $C_L(v) = \langle x \rangle$ ($r = 5$) and $N_L(Fv) \cong S_3$. This S_3 lies in a maximal subgroup of L of type $L_2(7)$. This implies that $C_G(v) \cong S_3$. The elements of order 6 of $L.2$ outside L have the property that there 3rd power belongs to the class $2B$ outside L. Since every extension of S_3 by a group of order 2 is isomorphic to $S_3 \times Z_2$, this shows that $C_{G_0}(v) = C_G(v)$."

148_{11} : Insert indices: $C_G(w_1)/C_G(W_1) \times \cdots \times C_G(w_n)/C_G(W_n)$.

150^{14} : Deleete one bracket: $T = N(EZ) \operatorname{wr} \mathrm{S_n}$

157_{16} : ... and we get a real vector in V as in the (*revised*) proof for Theorem 5.4.

159_{14} : A simplified argument is as follows: " Hence there are $|G/N|$ distinct G-conjugates of $\widetilde{V}_0 = \widetilde{U}_0 \otimes_F \widetilde{W}_0$ by Clifford's theorem (which are nonisomorphic F_0N-modules).

On the other hand, $\mathrm{Res}^{\widetilde{G}}_{\widetilde{N}}(V)$ has a unique irreducible constituent $\widetilde{V}$, which by (3) is absolutely irreducible. We conclude that $\widetilde{V}_0 \cong F_0 \otimes \widetilde{V}$ is the unique irreducible constituent of $\mathrm{Res}^{\widetilde{G}}_{\widetilde{N}}(F_0 \otimes_F V)$, and we obtain the contradiction $N = G$."

161_{13} : Replace 5.3b by 5.3a.

162^{16} : Add one bracket: $T = N_H(P)$.

187_{20} : Replace $k(Ya/Y_{11})$ by $k(Y_a/Y_{10})$ (in the table).

190^{3} : Numbers like 2.400 should read 2400 (?).

197_{3} : For $p = 2$ the congruences tell us nothing, however in this case the group G has odd order and so the Burnside congruence (1.5a) applies.

198^{2} : Just assert that $\chi(1) \equiv \pm 1 \pmod p$ and $|G| \equiv \pm 1 \pmod p$.

198_{12} : The proof should end with $|G| = |H| \cdot |G : H| \equiv \pm 1 \pmod p$.

198_{4} : The term $\chi_{v,\zeta}(1)$ should read $\chi_{v,\theta}(1)$.

200^{9} : Though not needed in the text, we mention that the same holds if $H = C_G(v)$ is just abelian. Use that the kernel of an FH-module V_i agrees with the kernel of its dual module V_i^* (and use the direct decomposition of HV).

232^{7} : "Spectral pattern, 76" should follow after "Singer cycle, 43, 195" and should be followed by "Splitting field, 2". (One "Spectral pattern" is enough.)