Florian Johne

Surgery for extended Ricci flow systems

We study extended Ricci flow systems in three dimensions. The extended system obtained by coupling Ricci flow to a function satisfying the heat equation is known as List flow in the literature. The motivation to study this system stems from its connection to static solutions to the vacuum Einstein equations and from its relation to four-dimensional Ricci flow with symmetry.

Our first result concerns the classification of singularity models of the extended system: The singularity models have non-negative sectional curvature.

The main ingredient in the proof is an improved bound on the Hessian along the flow.

The second contribution is a convergence result: In contrast to the Ricci flow, a round sphere may become less round under the flow initially. We identify conditions such that the metric converges to a metric of constant positive sectional curvature and the function to a constant. Our conditions reduce to R. Hamilton’s 1982 result in the limit of vanishing coupling.

The third contribution is an adaptation of the surgery procedure due to R. Hamilton and G. Perelman to the extended Ricci flow system. An important point in the construction is to show that the a-priori bound on the energy density is preserved through the surgery procedure.