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Limits of Spaces

Summer semester 2019 **15.04.2019**

Exercise sheet 1

Exercise 1

(Finite metric spaces - 2 points) Let $\mathcal{X}_{n,D} = \{(M,d) \,|\, |M| \le n, \operatorname{diam} M \le D\}$. Define a topology τ on $\mathcal{X}_n$ that makes $(\mathcal{X}_n, \tau)$ into a topological space.
Can you give a metric on $\mathcal{X}_{n,D}$ making it into a compact metric space? (+2 points)

Exercise 2

(Menger convexity - 2 points) Show that a complete metric space is geodesic if and only if for all distinct $x, y \in M$ there is a $z \in M\backslash\{x,y\}$ such that $d(x,z) + d(z,y) = d(x,y)$.

Exercise 3

(Hopf–Rinow - 3 points) Let (M,d) be a metric space. Show that the following statements are equivalent:

1. (M,d) is a proper length space
2. (M,d) is locally compact, geodesic and whenever $\gamma : [0,1) \to M$ is a geodesic then γ can be completed to a geodesic on all of $[0,1]$.

Exercise 4

(Branching geodesics - 3 points) Assume for four distinct points $x, y, z, m \in M$ it holds

$$d(x,m) + d(m,y) = d(x,y)$$
$$d(x,m) + d(m,z) = d(x,z).$$

Show that there are geodesics $\gamma, \eta : [0,1] \to M$ and a $t_0 \in (0,1)$ such that $\gamma_0 = \eta_0 = x$, $\gamma_1 = y$, $\eta_1 = z$ and

$$\gamma\big|_{[0,t_0]} \equiv \eta\big|_{[0,t_0]}.$$

Deadline 29.04.2019 in mailbox C-buidling 3rd floor or office C5P08.
You reach the website of the lecture under `http://tinyurl.com/Limits-of-Spaces`.