EBERHARD KARLS UNIVERSITÄT TÜBINGEN

Faculty of Science

Department of Mathematics

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Linear PDE

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Exercise sheet 7

Exercise 24
(3 points) Let $\alpha \in (0,1)$ and $B_1(\mathbf{0}) \subset \mathbb{R}^n$. Determine the range of p for which $u_\alpha \in W^{1,p}(B_1(\mathbf{0})$ where

$$u_\alpha(x) = \|x\|^\alpha.$$

(Hint: Recall that there is a constant $C = C(n,p)$ such that

$$C^{-1} \max_{i=1}^{n} |x_i| \le (\sum_{i=1}^{n} |x_i|^p)^{\frac{1}{p}} \le C \max_{i=1}^{n} |x_i|.$$

Use polar coordinates to estimate $(\sum_{i=1}^n |\partial_i u_\alpha|^p)^{\frac{1}{p}}$ pointwise on $B_1(\mathbf{0})\backslash\{\mathbf{0}\}$ from above and below and use cut-off functions to approximate u_α by C^1-functions.)

Exercise 25
(3 points) Let $s > 0$ and $B_1(\mathbf{0}) \subset \mathbb{R}^n$. Determine the range of p for which $u_s \in W^{1,p}(B_1(\mathbf{0}))$ where

$$u_s(x) = \frac{1}{\|x\|^s}.$$

(2 points *) In combination of the previous exercise, determine ranges for k and p such that

$$u_\alpha \in W^{k,p}(B_1(\mathbf{0})).$$

Exercise 26
Let $\Omega \subset \mathbb{R}^n$ be a bounded, convex domain and define $\tilde{\Omega} \subset \mathbb{R}^n \times \mathbb{R}$ by

$$\tilde{\Omega} = \{(x,t) \,|\, t \in (-1,1), \frac{x}{\sqrt{1-t^2}} \in \Omega\}.$$

(a) (2 points) The domain $\tilde{\Omega}$ is convex and if the boundary of Ω is C^k then the boundary $\partial\tilde{\Omega}$ is C^k (away from $(0,\pm 1) \in \partial\tilde{\Omega}$).

(b) (2 points) There is a natural injective embedding i of $W^{1,p}(\Omega)$ into $W^{1,p}(\tilde{\Omega})$ such that for each elliptic operator $L : C^2(\Omega) \to C^0(\Omega)$ be an elliptic operator there is an elliptic operator $\tilde{L} : C^2(\tilde{\Omega}) \to C^0(\tilde{\Omega})$ with the same bounds on the constant such that whenever $Lu = 0$ then $\tilde{L}\tilde{u} = 0$ where $\tilde{u} = i(u)$ is given by the embedding above.

Deadline 19.06.2016 in before the lecture.
You reach the website of the lecture under `https://tinyurl.com/UniTue-LinPDE`.