Title: *The $\mathbb{Q}$-algebraicity problem in real algebraic geometry*

Abstract. In 2020, Parusiński and Rond proved that every algebraic set $V \subset \mathbb{R}^n$ is homeomorphic to a $\overline{\mathbb{Q}}^r$-algebraic set $V' \subset \mathbb{R}^n$, where $\overline{\mathbb{Q}}^r$ denotes the field of real algebraic numbers. The aim of this talk is provide some classes of algebraic sets that positively answer the following open problem:

$\mathbb{Q}$-ALGEBRAICITY PROBLEM: (Parusiński, 2021) Is every algebraic set $V \subset \mathbb{R}^n$ homeomorphic to some $\mathbb{Q}$-algebraic set $V' \subset \mathbb{R}^m$, with $m \geqslant n$?

We recall the notions of $\mathbb{R}|\mathbb{Q}$-local ring, $\mathbb{R}|\mathbb{Q}$-singular and $\mathbb{R}|\mathbb{Q}$-nonsingular points for a $\mathbb{Q}$-algebraic set introduced by Fernando and Ghiloni. Hence, a natural notion of $\mathbb{Q}$-nonsingular $\mathbb{Q}$-algebraic set arise. After an overview on approximation results in real algebraic geometry, I will explain how $\mathbb{Q}$-nonsingular $\mathbb{Q}$-algebraic sets come into play to provide a version over $\mathbb{Q}$ of the relative Nash-Tognoli theorem. Latter result, combined with resolution of singularities and a version over $\mathbb{Q}$ of the classical blowing down lemma, allows us to give a complete positive answer to the above $\mathbb{Q}$-ALGEBRAICITY PROBLEM in the case of nonsingular algebraic sets and algebraic sets with isolated singularities.