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Fachbereich Mathematik
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Algebraic Number Theory
SS 2026

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Sheet 7

If you want your solutions to be corrected, upload them on URM by the end of June 15.
Write your name and matriculation number on each sheet. If you upload the exercises as a group, only one person can do it instead of the whole group.
The exercises marked with $*$ might be more complicated or involved than the others.

Exercise 7.1 Let A be a Dedekind domain and let $I, J \subseteq A$ be two non-zero ideals such that $I^m = J^m$ for a certain $m > 0$.

(i) Show that $I = J$.

(ii) ($*$) Is the statement true if A is a Noetherian domain but not Dedekind? Prove it or give a counterexample.

Exercise 7.2 Let K be a number field and $I \subseteq \mathcal{O}_K$ an ideal.

(i) Show that there exist $m \in \mathbf{Z}_{>0}$ such that I^m is a principal ideal.

(ii) Show that there exists a finite extension L of K such that I becomes principal in $\mathcal{O}_L$, i.e. $I \cdot \mathcal{O}_L = (\beta)$ for some $\beta \in \mathcal{O}_L$.

(iii) Show that there exists a finite extension E of K where every ideal of $\mathcal{O}_K$ becomes principal: $I \cdot \mathcal{O}_E$ is principal for every ideal $I \subseteq \mathcal{O}_K$.

Exercise 7.3 Compute the ideal class number of the following number fields:

$$\mathbf{Q}(\sqrt{-3}),\ \mathbf{Q}(\sqrt{-7}),\ \mathbf{Q}(\sqrt{-11}),\ \mathbf{Q}(\zeta_5).$$