



Sheet 8

If you want your solutions to be corrected, upload them on URM by the end of June 22.
Write your name and matriculation number on each sheet. If you upload the exercises as a group,
only one person can do it instead of the whole group.

The exercises marked with * might be more complicated or involved than the others.

Exercise 8.1 Show that the class group of the field $\mathbf{Q}(\sqrt{10})$ is isomorphic to $\mathbf{Z}/2\mathbf{Z}$ and find an explicit generator.

Exercise 8.2 Let K be a number field and let $\sigma_1, \dots, \sigma_n: K \hookrightarrow \mathbf{C}$ be all the embeddings of K . In the lectures we saw that if $\alpha \in \mathcal{O}_K$ is a root of unity if and only if $|\sigma_i(\alpha)| = 1$ for all i .

- (i) Let $\alpha \in K$ be such that $|\sigma_i(\alpha)| = 1$ for all i . Is it true that α is a root of unity? Prove it or give a counterexample.
- (ii) Let $\alpha \in \mathcal{O}_K$ be such that $|\alpha| = 1$. Is it true that α is a root of unity? Prove it or give a counterexample.