Eberhard-Karls-Universität Tübingen
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20. Januar 2021

Introduction to Commutative Algebra and Algebraic Geometry
Exercise Sheet 10

Exercise 1.
Let K be an algebraically closed field and let X be an affine variety. Let $f : X \to K$ be a map. Prove that the following statements are equivalent:

(i) $f \in \mathcal{O}_X(X)$.

(ii) $f : X \to K$ is a morphism.

Exercise 2.
Let X be an affine variety and $\mathcal{F}$ its sheaf of regular functions. Let $U \subset X$ be an open set.
Let $s \in \mathcal{F}(U)$ be an element with $s_x = 0 \in \mathcal{F}_x$ for all $x \in U$. Show: $s = 0$.

Exercise 3.
Let K be an algebraically closed field with $\mathrm{char}(K) = 0$. Consider the map from the affine line $\mathbb{A}^1_K$ to the curve $C = V(y^2 - x^3)$ given by $\phi : \mathbb{A}^1_K \to C, t \mapsto (t^2, t^3)$. Prove ϕ is a morphism and a homeomorphism (i.e. bijective, continuous and open).

Submission: In groups of up to three students, until Wednesday, 27. January 2021, 12:00 o'clock via URM. You are allowed to submit your solutions in German.