Eberhard-Karls-Universität Tübingen
Prof. Hannah Markwig
Alheydis Geiger

Wintersemester 2020/2021
3. February 2021

Introduction to Commutative Algebra and Algebraic Geometry
Exercise Sheet 12

Exercise 1.
Let K be an algebraically closed field, $f, g \in K[x_1, \dots, x_n]$, $V(f, g) \subset K^n$. Prove the equivalence of the following statements:

(i) $\mathsf{gcd}(f, g) = 1$.

(ii) $\dim(V(f, g)) \leq n - 2$.

Exercise 2.
Show that the statement from Theorem 4.2.1 2) does not generally hold if K is not algebraically closed.
For this consider $K = \mathbb{R}$ and $X = Y = K$ and $\varphi : X \to Y$, $t \mapsto t^2$ and the ideal $J = \langle x^2 + 1 \rangle \subset K[x]$.

Exercise 3.
Determine the number of irreducible components, the dimension and the ring of global functions for every fibre of the following morphisms:

(i) $\varphi : K^2 \to K, (z, w) \mapsto zw,$

(ii) $\psi : K^2 \to K^2, (z, w) \mapsto (zw, w).$

Submission: In groups of up to three students, until Wednesday, 10. February 2021, 12:00 o'clock via URM. You are allowed to submit your solutions in German.