Eberhard-Karls-Universität Tübingen
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10. February 2021

# Introduction to Commutative Algebra and Algebraic Geometry
# Exercise Sheet 13

**Exercise 1.**
Consider $X := Y := K^2$ and the morphism $\varphi : X \to Y$, $(z, w) \mapsto (zw, w)$. Prove:

(i) $\varphi$ is birational, but not an isomorphism.

(ii) For $g(a, b) = \frac{a}{b}$ we have $g \in K(Y) \setminus \mathcal{O}(Y)$ but $\varphi^*(g) \in \mathcal{O}(X)$.

**Exercise 2.**
Let $K$ be an algebraically closed field and $X = V(x^3 - y^2) \subset K^2$. Prove: the coordinate ring $K[X]$ is not normal.

**Exercise 3.**
Let $R \subset S$ and $S \subset T$ be integral ring extensions. Prove that $R \subset T$ is an integral ring extension.

**Submission:** In groups of up to three students, until Wednesday, 10. February 2021, 12:00 o'clock via URM. You are allowed to submit your solutions in German.