Eberhard-Karls-Universität Tübingen
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Introduction to Commutative Algebra and Algebraic Geometry
Exercise Sheet 4

Exercise 1.
Let X be a noetherian topological space and let $X = X_1 \cup \ldots \cup X_n$ be the decomposition into irreducible components. Show: If $U \subset X$ is a non-empty open subset, then the irreducible components of U are exactly the sets $X_i \cap U$ with $i = 1, \ldots, n$ for which $X_i \cap U \neq \emptyset$.

Exercise 2.

a) Let $\varphi : X \to Y$ be a continuous map of noetherian topological spaces and let $Z \subset Y$ be the closure of the image $\varphi(X)$. Prove: The maximal number of irreducible components in the minimal decomposition of Z is bounded by the number of irreducible components of the minimal decomposition of X.

b) Let $X = V(xy + x - y - 1) \subset \mathbb{C}^2$, $Y = \mathbb{C}$ and let $\varphi : X \to Y$ be defined by $\varphi((a, b)) = a$. Prove that X and Y are noetherian topological spaces, that φ is continuous with respect to the Zariski topology and that the number of irreducible components of the closure of $\varphi(X)$ is strictly smaller than the number of irreducible components of X.

Exercise 3.
Let K be a field and let $X := \{A \in \mathrm{Mat}(n \times n; K) | \ \mathrm{rank}(A) \leq 1\}$. Prove: X is irreducible in $\mathrm{Mat}(n \times n; K) = K^{n \times n}$.

Submission: In groups of up to three students, until Wednesday, 2. December 2020, 12:00 o'clock via URM. You are allowed to submit your solutions in German.