Eberhard-Karls-Universität Tübingen
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2. December 2020

Introduction to Commutative Algebra and Algebraic Geometry
Exercise Sheet 5

Exercise 1.
Let $>$ be a monomial order on Mon_n. Prove that the following are equivalent:

1) $>$ is global.
2) $>$ is a well-ordering
3) $x_i > 1 \; \forall\, i = 1, \ldots, n$
4) $>$ refines $\geq_{\mathsf{nat}}$, i.e. $\underline{x}^{\alpha} >_{\mathsf{nat}} \underline{x}^{\beta} \;\Rightarrow\; \underline{x}^{\alpha} > \underline{x}^{\beta}$

Exercise 2. a) Prove that the set of monomials of $K[X]$ has exactly two orders, one of which is global.

b) Prove that the set of monomials of $K[X, Y]$ has uncountable many orders.
Hint: Consider weighted degree reverse lexicographic orders.

Exercise 3.
Let K be an algebraically closed field. Let $A \subset K^n$ be an affine varieties and let $f : A \to K^m$, $a \mapsto (f_1(a), \ldots, f_m(a))$ be a polynomial map, so $f_1, \ldots, f_m \in K[X_1, \ldots, X_n]$. We write $K[X_1, ..., X_n]$ for the coordinate ring to K^n and $K[Y_1, ..., Y_m]$ for the coordinate ring to K^m.
Prove: $\overline{f(A)} = V(\langle I(A), Y_1 - f_1, \ldots, Y_m - f_m\rangle_{K[X_1,\ldots,X_n,Y_1,\ldots,Y_m]} \cap K[\underline{Y}])$.

Submission: In groups of up to three students, until Wednesday, 9. December 2020, 12:00 o'clock via URM. You are allowed to submit your solutions in German.