Eberhard-Karls-Universität Tübingen
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Wintersemester 2020/2021
16. December 2020

Introduction to Commutative Algebra and Algebraic Geometry
Exercise Sheet 7

Exercise 1.
Let $R = \mathbb{Q}[x, y, z]$. Compute a Groebner basis of $I = \langle xy - y, 2x^2 + yz, y - z\rangle \subset R$ with respect to the monomial order $>_{lp}$ with $x > z > y$.

Exercise 2.
Let $I \subset K[\underline{x}]$ be an ideal, $>$ a global monomial ordering and $B = \text{Mon}(\underline{x}) \cap (K[\underline{x}] \setminus L(I))$ the set of monomials which are not in $L(I)$ the leading ideal of I. Show that $\overline{B}$ is a K-vector space basis of $K[\underline{x}]/I$.

Exercise 3.
Show that

a) the Groebner basis algorithm (2.3.8.) coincides with the *Euclidean algorithm* when applied to two polynomials in $K[t]$ with $>$ being the unique well-ordering on $K[t]$;

b) the reduced Groebner basis algorithm (2.3.9.) coincides with the *Gaussian algorithm* when applied to any finite list of linear polynomials in $K[x_1, \ldots, x_n]$ with $>$ being the degree lexicographic ordering.

Recall: The degree lexicographical ordering $>_{Dp}$ on Mon$_n$ is defined by

$$\underline{x}^\alpha >_{Dp} \underline{x}^\beta :\Leftrightarrow |\alpha| > |\beta| \text{ or } (|\alpha| = |\beta| \text{ and } \exists k : \alpha_1 = \beta_1, \ldots, \alpha_{k-1} = \beta_{k-1}, \alpha_k > \beta_k).$$

Submission: In groups of up to three students, until Wednesday, 23. December 2020, 12:00 o'clock via URM. You are allowed to submit your solutions in German.