Eberhard-Karls-Universität Tübingen
Prof. Hannah Markwig
Alheydis Geiger

Wintersemester 2020/2021
23. December 2020

Introduction to Commutative Algebra and Algebraic Geometry
Exercise Sheet 8

Exercise 1.
Let $f, g \in K[x]$, the polynomial ring in *one* variable. Express the greatest common divisor and the least common multiple of f and g in terms of elements in $\mathrm{syz}(f, g)$ and derive an algorithm to compute these, assuming we can compute a Gröbner basis of $\mathrm{syz}(f, g)$.

Exercise 2.
Let

$$V = V((x + y) \cdot (x - y) \cdot (x + z^2)) \subset \mathbb{C}^3$$

and

$$W = V((x + z^2) \cdot (x - y) \cdot (z + y)) \subset \mathbb{C}^3.$$

Draw sketches of V and W in $\mathbb{R}^3$, and try to guess the ideal of $\overline{V \setminus W}$. How could you verify your guess? Answer with an approriate algorithm and describe the steps accurately without doing all the computations.

Exercise 3. a) Is the polynomial $f = x^7 + x^2$ in the radical of $I = \langle x^{11} + x^6, yx^4 + x^8, y^3 + x^2 \rangle$? Use algorithm 2.5.5 to prove your claim.

Hint: Eliminate y first.

b) Let $I = \langle x^2 + 2y^2 - 3, x^2 + xy + y^2 - 3 \rangle \subset \mathbb{Q}[x, y]$. Compute $I \cap \mathbb{Q}[y]$.

Submission: In groups of up to three students, until Wednesday, 13. January 2021, 12:00 o'clock via URM. You are allowed to submit your solutions in German.