

Introduction to Commutative Algebra and Algebraic Geometry

Exercise Sheet 9

Exercise 1.

Let K be an algebraically closed field, $A := K[T_1, T_2]/\langle T_1 \cdot T_2 \rangle$ and $f \in A$ the equivalence class of T_1 . Prove: $A_f \cong K[T]_T$.

Exercise 2.

Let R be a commutative ring with one and let $S \subset R$ be multiplicatively closed. Show: If R is noetherian, then $S^{-1}R$ is noetherian.

Exercise 3.

Let K be an algebraically closed field. Determine the algebra of regular functions $\mathcal{O}(U_i)$ for the following open sets U_i :

- $U_1 = K \setminus \{0\} \subset K$,
- $U_2 = K^\times \times K \subset K^2$,
- $U_3 = K^\times \times K^\times \subset K^2$.